# COMP 3361 Natural Language Processing

## Lecture 5: Word Embeddings (cont'd)

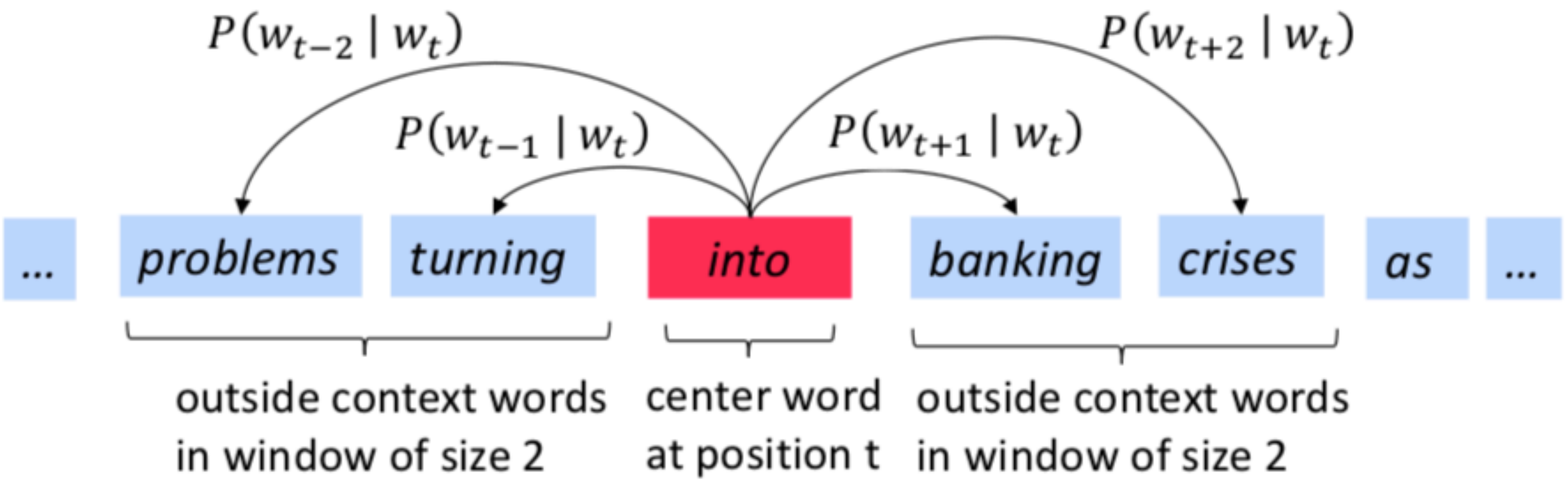
Spring 2024

Many materials from CS224n@Stanford and COS484@Princeton with special thanks!

# Skip-gram

- Assume that we have a large corpus $w_1, w_2, \ldots, w_T \in V$
- **Key idea:** Use each word to **predict** other words in its context
- Context: a fixed window of size 2*m* (m = 2 in the example)

A classification problem!

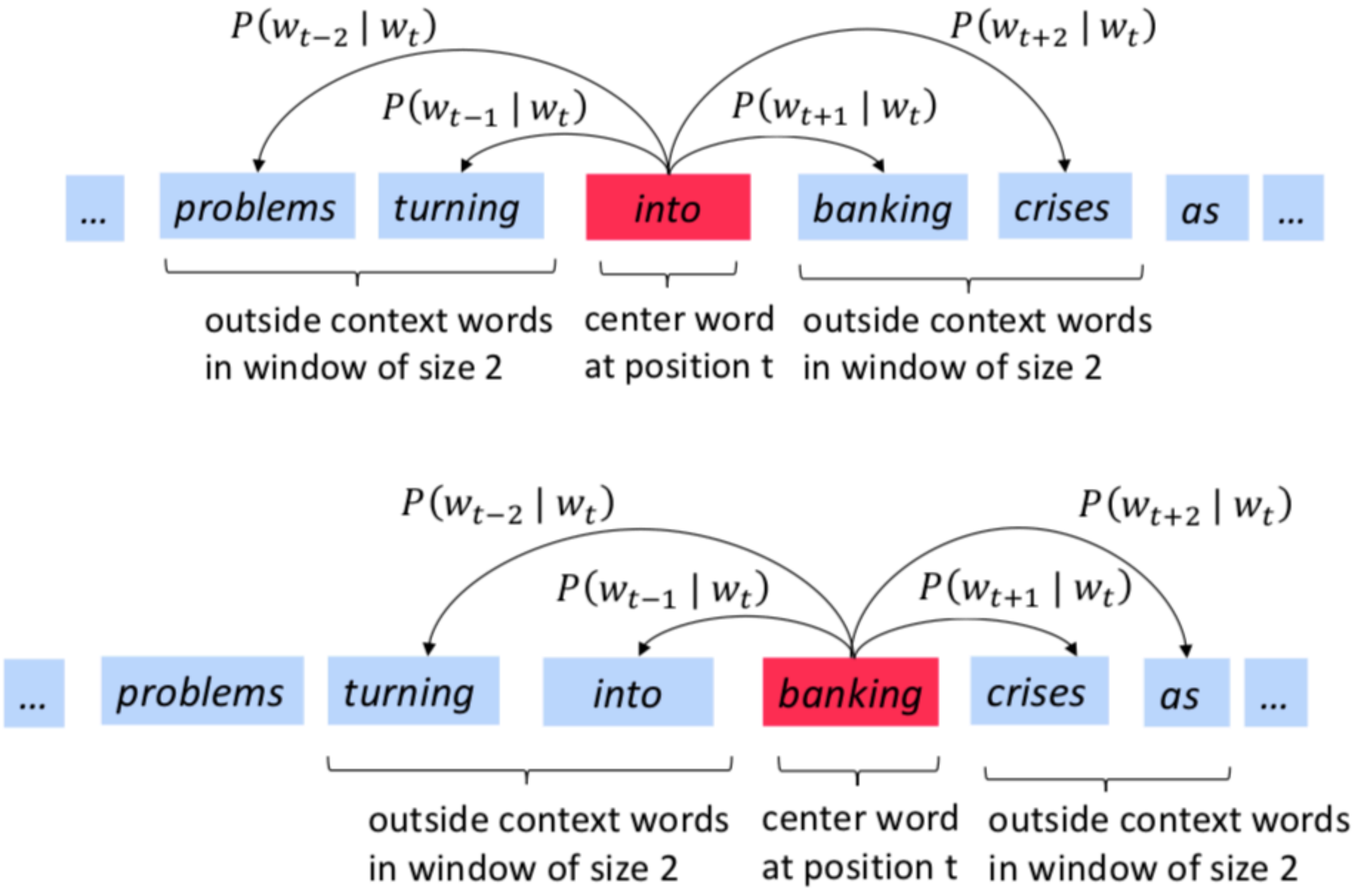


$P(b \mid a)$ = given the center word is $a$, what is the probability that $b$ is a context word?

$P(\cdot \mid a)$ is a probability distribution defined over V: $\sum_{w \in V} P(w \mid a) = 1$

We are going to define this distribution soon!

# Skip-gram



Convert the training data into:

(into, problems)
(into, turning)
(into, banking)
(into, crises)
(banking, turning)
(banking, into)
(banking, crises)
(banking, as)
…

Our goal is to find parameters that can maximize

$$\underline{P(\text{problems} \mid \text{into}) \times P(\text{turning} \mid \text{into}) \times P(\text{banking} \mid \text{into}) \times P(\text{crises} \mid \text{into})} \times \underline{P(\text{turning} \mid \text{banking}) \times P(\text{into} \mid \text{banking}) \times P(\text{crises} \mid \text{banking}) \times P(\text{as} \mid \text{banking})}\ldots$$

# Skip-gram: objective function

- For each position $t = 1,2,\ldots T$, predict context words within context size m, given center word $w_t$:

all the parameters to be optimized

$$\mathcal{L}(\theta) = \prod_{t=1}^{T} \prod_{-m \le j \le m, j \ne 0} P(w_{t+j} \mid w_t; \theta)$$

- It is equivalent as minimizing the (average) negative log likelihood:

$$J(\theta) = -\frac{1}{T} \log \mathcal{L}(\theta) = -\frac{1}{T} \sum_{t=1}^{T} \sum_{-m \le j \le m, j \ne 0} \log P(w_{t+j} \mid w_t; \theta)$$

# How to define $P(w_{t+j} \mid w_t; \theta)$?

- Use two sets of vectors for each word in the vocabulary

$$\mathbf{u}_a \in \mathbb{R}^d\text{: vector for center word } a,\ \forall a \in V$$

$$\mathbf{v}_b \in \mathbb{R}^d\text{: vector for context word } b,\ \forall b \in V$$

- Use inner product $\mathbf{u}_a \cdot \mathbf{v}_b$ to measure how likely word *a* appears with context word *b*

Softmax we have seen in multinomial logistic regression!

$$P(w_{t+j} \mid w_t) = \frac{\exp(\mathbf{u}_{w_t} \cdot \mathbf{v}_{w_{t+j}})}{\sum_{k \in V} \exp(\mathbf{u}_{w_t} \cdot \mathbf{v}_k)}$$

Recall that $P(\cdot \mid a)$ is a probability distribution defined over V…

# Important note

$$J(\theta) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{-m\leq j\leq m, j\neq 0}\log\frac{\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_{w_{t+j}})}{\sum_{k\in V}\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_k)}$$

- In this formulation, we don’t care about the classification task itself like we do for the logistic regression model we saw previously.

- The key point is that the *parameters* used to optimize this training objective—when the training corpus is large enough—can give us very good representations of words (following the principle of distributional hypothesis)!

# How many parameters in this model?

$$J(\theta) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{-m\le j\le m, j\ne 0}\log\frac{\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_{w_{t+j}})}{\sum_{k\in V}\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_k)}$$

How many parameters does this model have (i.e. what is size of $\theta$)?

(a) $d|V|$

(b) $2d|V|$

(c) $2m|V|$

(d) $2md|V|$

d = dimension of each vector

The answer is (b).

Each word has two d-dimensional vectors, so it is $2 \times |V| \times d$.

# word2vec formulation

$$J(\theta) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{-m\leq j\leq m, j\neq 0}\log\frac{\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_{w_{t+j}})}{\sum_{k\in V}\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_k)}$$

Q: Why do we need two vectors for each word instead of one?

A: because one word is not likely to appear in its own context window, e.g., $P$(dog | dog) should be low. If we use one set of vectors only, it essentially needs to minimize $\mathbf{u}_{\text{dog}} \cdot \mathbf{u}_{\text{dog}}$..

Q: Which set of vectors are used as word embeddings?

A: This is an empirical question. Typically just $\mathbf{u}_w$ but you can also concatenate the two vectors..

# How to train this model?

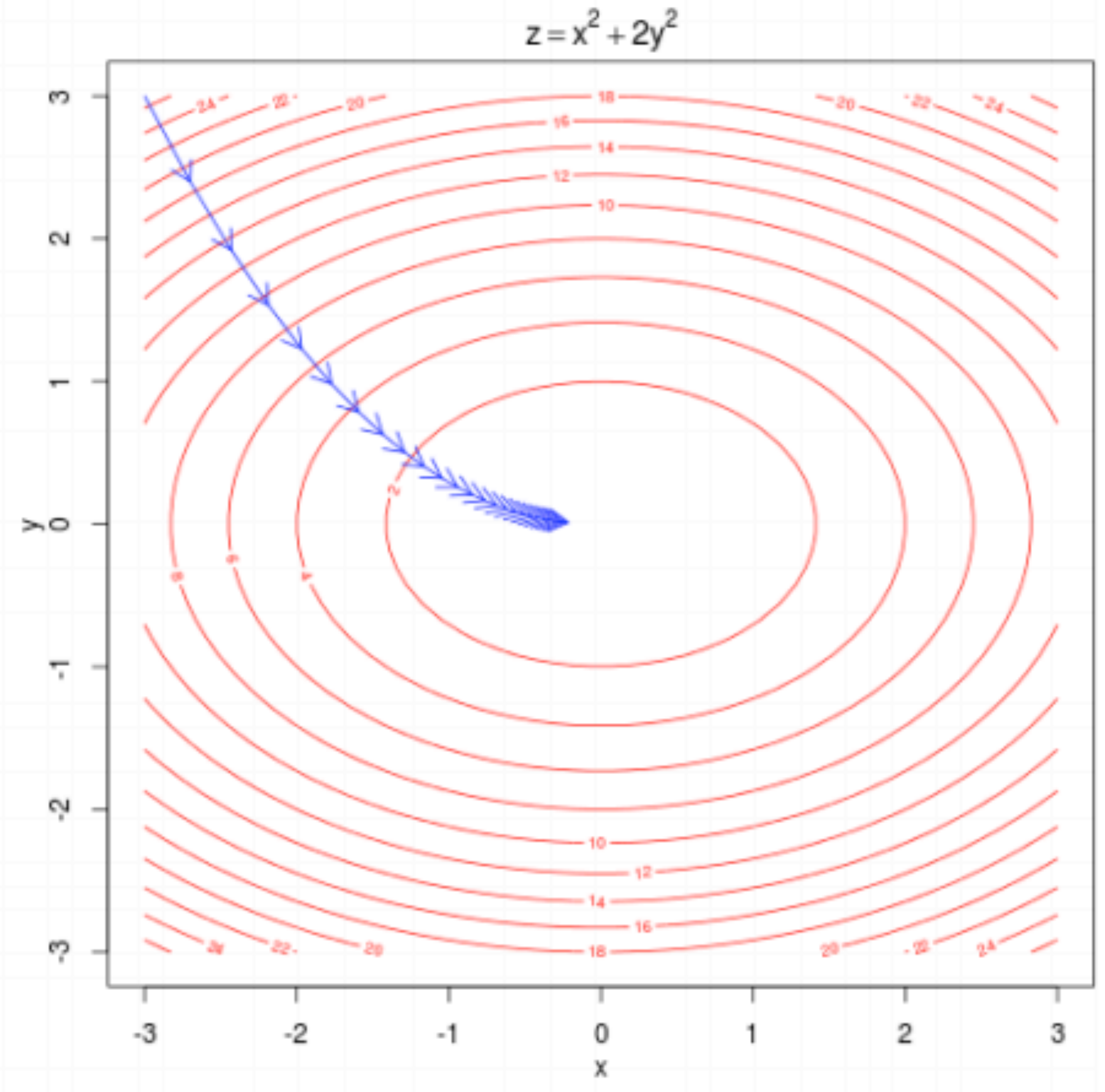
$$J(\theta) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{-m\le j\le m, j\ne 0}\log\frac{\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_{w_{t+j}})}{\sum_{k\in V}\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_k)}$$

- To train such a model, we can use stochastic gradient descent to optimize $\theta$

- Again, $\theta$ represents all $2d|V|$ model parameters, in one vector.

- To train such a model, we need to compute the vector gradient $\nabla_\theta J(\theta) = ?$

$$\theta = \begin{bmatrix} v_{aardvark} \\ v_a \\ \vdots \\ v_{zebra} \\ u_{aardvark} \\ u_a \\ \vdots \\ u_{zebra} \end{bmatrix}$$



# Vectorized gradients

$$f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{a}$$

$$\mathbf{x}, \mathbf{a} \in \mathbb{R}^n$$

$$\frac{\partial f}{\partial \mathbf{x}} = \mathbf{a}$$

$$f = x_1 a_1 + x_2 a_2 + \ldots + x_n a_n$$

$$\frac{\partial f}{\partial \mathbf{x}} = [\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}]$$

# Vectorized gradients: exercises

Let $f = \exp(\mathbf{w} \cdot \mathbf{x})$, what is the value of $\dfrac{\partial f}{\partial \mathbf{x}}$? $\mathbf{w}, \mathbf{x} \in \mathbb{R}^n$

(a) $\mathbf{w}$

(b) $\exp(\mathbf{w} \cdot \mathbf{x})$

(c) $\exp(\mathbf{w} \cdot \mathbf{x})\mathbf{w}$

(d) $\mathbf{x}$

The answer is (c).

$$\frac{\partial}{\partial x_i} = \frac{\exp(\sum_{k=1}^{n} w_i x_i)}{\partial x_i} = \exp(\sum_{k=1}^{n} w_i x_i) w_i$$

# Let’s compute gradients for word2vec

$$J(\theta) = -\frac{1}{T}\sum_{t=1}^{T}\sum_{-m\le j\le m, j\ne 0}\log\frac{\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_{w_{t+j}})}{\sum_{k\in V}\exp(\mathbf{u}_{w_t}\cdot\mathbf{v}_k)}$$

Consider one pair of center/context words $(t, c)$:

$$y = -\log\left(\frac{\exp(\mathbf{u}_t\cdot\mathbf{v}_c)}{\sum_{k\in V}\exp(\mathbf{u}_t\cdot\mathbf{v}_k)}\right)$$

We need to compute the gradient of $y$ with respect to

$\mathbf{u}_t$ and $\mathbf{v}_k, \forall k \in V$

# Let's compute gradients for word2vec

$$y = -\log\left(\frac{\exp(\mathbf{u}_t \cdot \mathbf{v}_c)}{\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k)}\right)$$

$$y = -\log(\exp(\mathbf{u}_t \cdot \mathbf{v}_c)) + \log(\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k))$$

$$= -\mathbf{u}_t \cdot \mathbf{v}_c + \log(\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k))$$

Recall that

$$P(w_{t+j} \mid w_t) = \frac{\exp(\mathbf{u}_{w_t} \cdot \mathbf{v}_{w_{t+j}})}{\sum_{k \in V} \exp(\mathbf{u}_{w_t} \cdot \mathbf{v}_k)}$$

$$\frac{\partial y}{\partial \mathbf{u}_t} = \frac{\partial(-\mathbf{u}_t \cdot \mathbf{v}_c)}{\partial \mathbf{u}_t} + \frac{\partial(\log \sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k))}{\partial \mathbf{u}_t}$$

$$= -\mathbf{v}_c + \frac{\frac{\partial \sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k)}{\partial \mathbf{u}_t}}{\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k)}$$

$$= -\mathbf{v}_c + \frac{\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k) \cdot \mathbf{v}_k}{\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k)}$$

$$\boxed{= -\mathbf{v}_c + \sum_{k \in V} \frac{\exp(\mathbf{u}_t \cdot \mathbf{v}_k)}{\sum_{k' \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_{k'})} \mathbf{v}_k}$$

$$= -\mathbf{v}_c + \sum_{k \in V} P(k \mid t) \mathbf{v}_k$$

# Gradients for word2vec

What about context vectors?

$$\frac{\partial y}{\partial \mathbf{v}_k} = \begin{cases} \left(P(k \mid t) - 1\right)\mathbf{u}_t & k = c \\ P(k \mid t)\mathbf{u}_t & k \neq c \end{cases}$$

$$y = -\log\left(\frac{\exp(\mathbf{u}_t \cdot \mathbf{v}_c)}{\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k)}\right)$$

Do it yourself :)

# Overall algorithm

Convert the training data into:
(into, problems)
(into, turning)
(into, banking)
(into, crises)
(banking, turning)
(banking, into)
(banking, crises)
(banking, as)
…

- Input: text corpus, embedding size $d$, vocabulary $V$, **context size m**
- Initialize $\mathbf{u}_i, \mathbf{v}_i$ randomly $\forall i \in V$
- Run through the training corpus and for each training instance (*t*, *c*):
  - Update $\mathbf{u}_t \leftarrow \mathbf{u}_t - \eta \frac{\partial y}{\partial \mathbf{u_t}}$ $\qquad \frac{\partial y}{\partial \mathbf{u}_t} = -\mathbf{v}_c + \sum_{k \in V} P(k \mid t)\mathbf{v}_k$
  - Update $\mathbf{v}_k \leftarrow \mathbf{v}_k - \eta \frac{\partial y}{\partial \mathbf{v_k}}, \forall k \in V$ $\qquad \frac{\partial y}{\partial \mathbf{v}_k} = \begin{cases} (P(k \mid t) - 1)\,\mathbf{u}_t & k = c \\ P(k \mid t)\mathbf{u}_t & k \neq c \end{cases}$

Q: Can you think of any issues with this algorithm?

# Skip-gram with negative sampling (SGNS)

Problem: every time you get one pair of ($t$, $c$), you need to update $\mathbf{v}_k$ with all the words in the vocabulary! This is very expensive computationally.

$$\frac{\partial y}{\partial \mathbf{u}_t} = -\mathbf{v}_c + \sum_{k \in V} P(k \mid t)\mathbf{v}_k \qquad \frac{\partial y}{\partial \mathbf{v}_k} = \begin{cases} (P(k \mid t) - 1)\,\mathbf{u}_t & k = c \\ P(k \mid t)\mathbf{u}_t & k \neq c \end{cases}$$

**Negative sampling**: instead of considering all the words in V, let's randomly sample $K$ (5-20) negative examples.

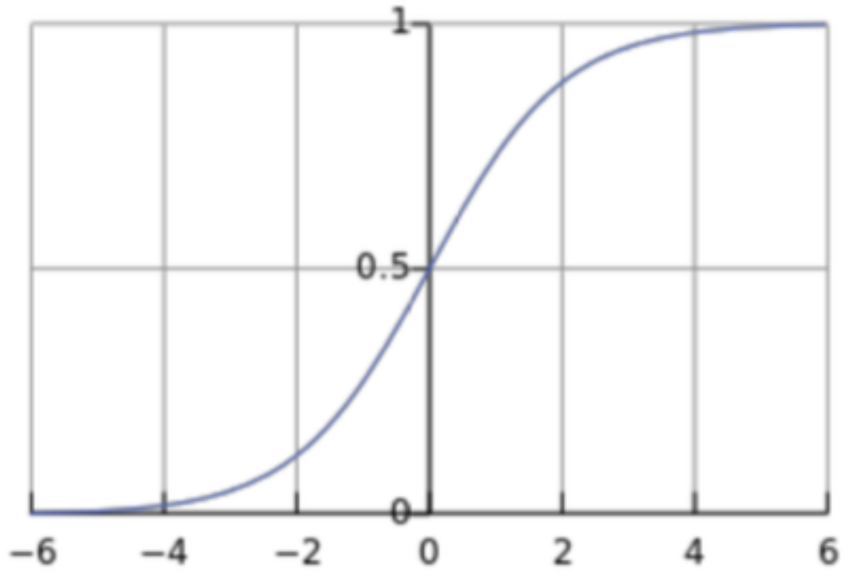
softmax:

$$y = -\log\left(\frac{\exp(\mathbf{u}_t \cdot \mathbf{v}_c)}{\sum_{k \in V} \exp(\mathbf{u}_t \cdot \mathbf{v}_k)}\right)$$

Negative sampling:

$$y = -\log(\sigma(\mathbf{u}_t \cdot \mathbf{v}_c)) - \sum_{i=1}^{K} \mathbb{E}_{j \sim P(w)} \log(\sigma(-\mathbf{u}_t \cdot \mathbf{v}_j))$$

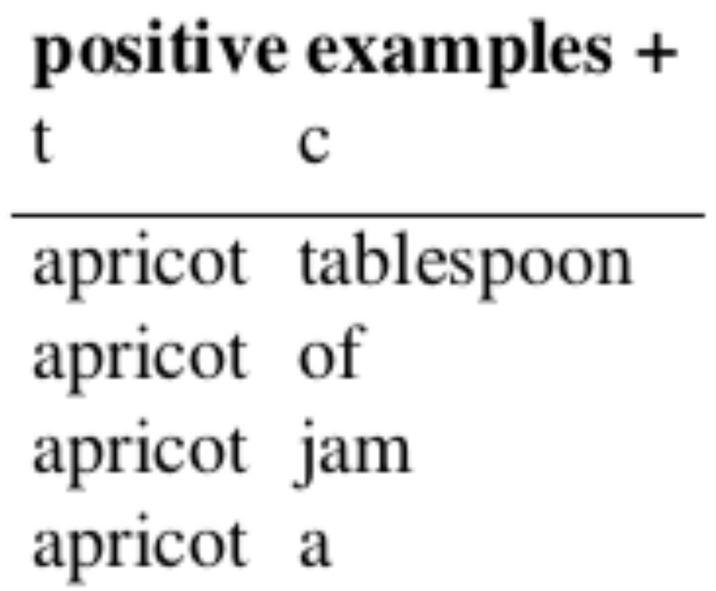
$$\sigma(x) = \frac{1}{1 + \exp(-x)}$$



# Skip-gram with negative sampling (SGNS)

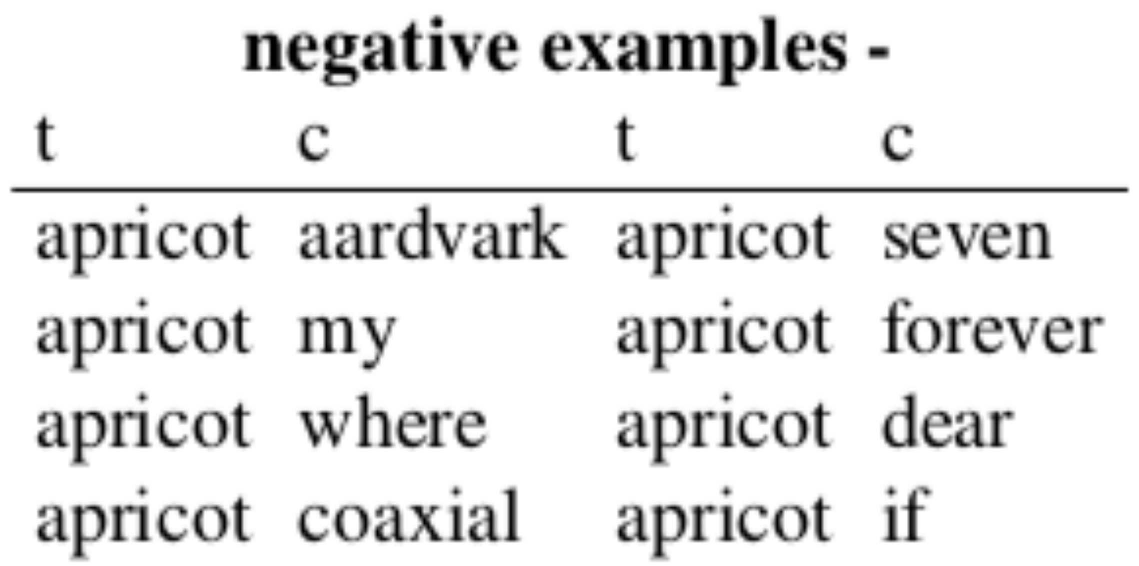
**Key idea: Convert the $|V|$-way classification into a set of binary classification tasks.**

Every time we get a pair of words (*t*, *c*), we don't predict *c* among all the words in the vocabulary. Instead, we predict (*t*, *c*) is a positive pair, and (*t*, *c'*) is a negative pair for a small number of sampled *c'*.



**positive examples +**

| t | c |
|---|---|
| apricot | tablespoon |
| apricot | of |
| apricot | jam |
| apricot | a |



**negative examples -**

| t | c | t | c |
|---|---|---|---|
| apricot | aardvark | apricot | seven |
| apricot | my | apricot | forever |
| apricot | where | apricot | dear |
| apricot | coaxial | apricot | if |

$$y = -\log(\sigma(\mathbf{u}_t \cdot \mathbf{v}_c)) - \sum_{i=1}^{K} \mathbb{E}_{j \sim P(w)} \log(\sigma(-\mathbf{u}_t \cdot \mathbf{v}_j))$$

P(w): sampling according to the frequency of words

Similar to **binary logistic regression**, but we need to optimize $\mathbf{u}$ and $\mathbf{v}$ together.

$$P(y = 1 \mid t, c) = \sigma(\mathbf{u}_t \cdot \mathbf{v}_c) \qquad p(y = 0 \mid t, c') = 1 - \sigma(\mathbf{u}_t \cdot \mathbf{v}_{c'}) = \sigma(-\mathbf{u}_t \cdot \mathbf{v}_{c'})$$

# Understanding SGNS

$$y = -\log(\sigma(\mathbf{u}_t \cdot \mathbf{v}_c)) - \sum_{i=1}^{K} \mathbb{E}_{j \sim P(w)} \log(\sigma(-\mathbf{u}_t \cdot \mathbf{v}_j))$$

In skip-gram with negative sampling (SGNS), how many parameters need to be updated in $\theta$ for every ($t$, $c$) pair?

(a) $Kd$

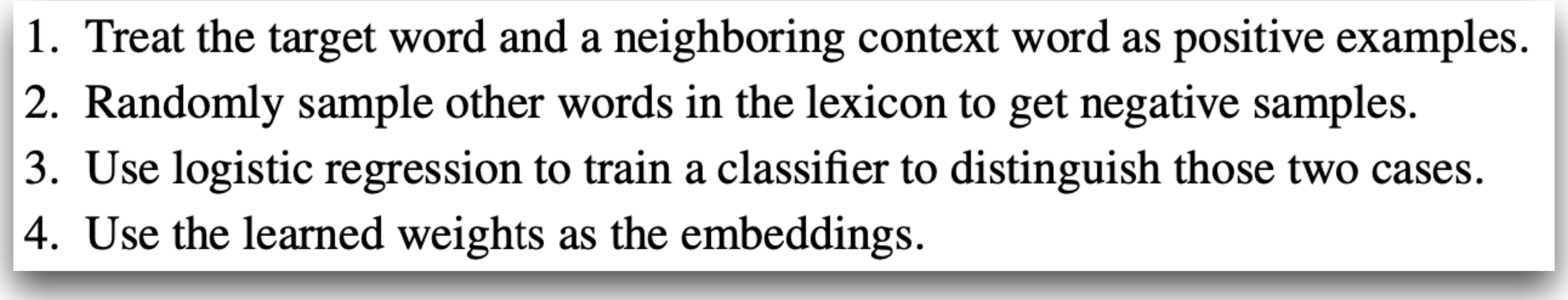
(b) $2Kd$

(c) $(K+1)d$

(d) $(K+2)d$

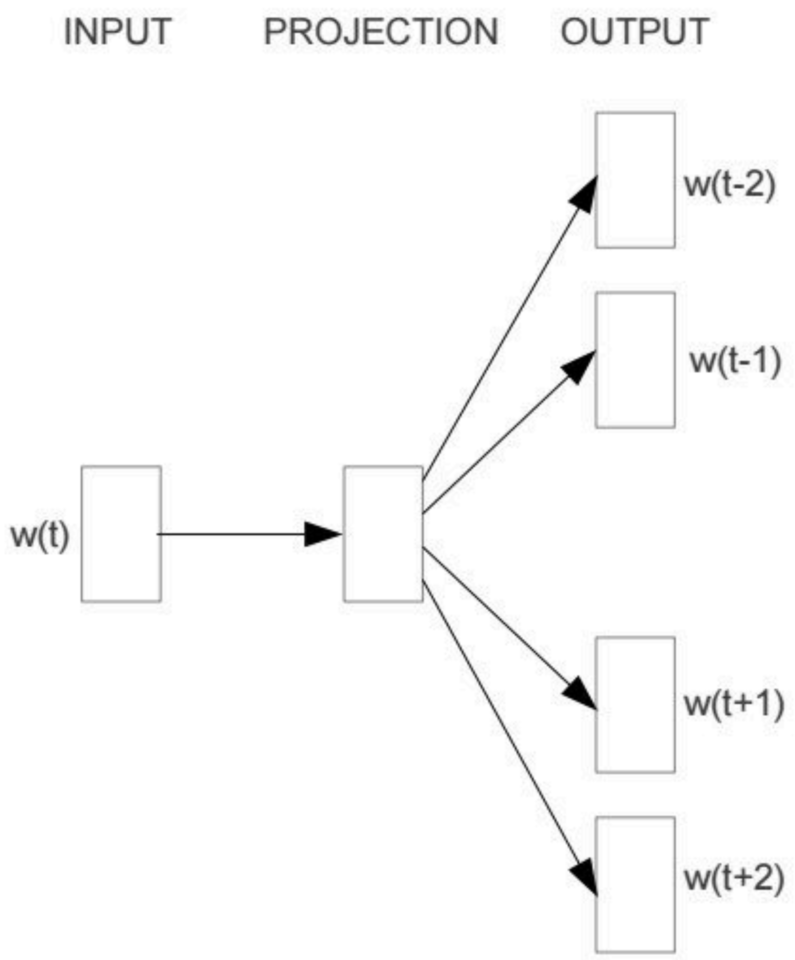
The answer is (d).
We need to calculate gradients with respect to $\mathbf{u}_t$ and (K + 1) $\mathbf{v}_i$ (one positive and K negatives).

# The intuition of SGNS

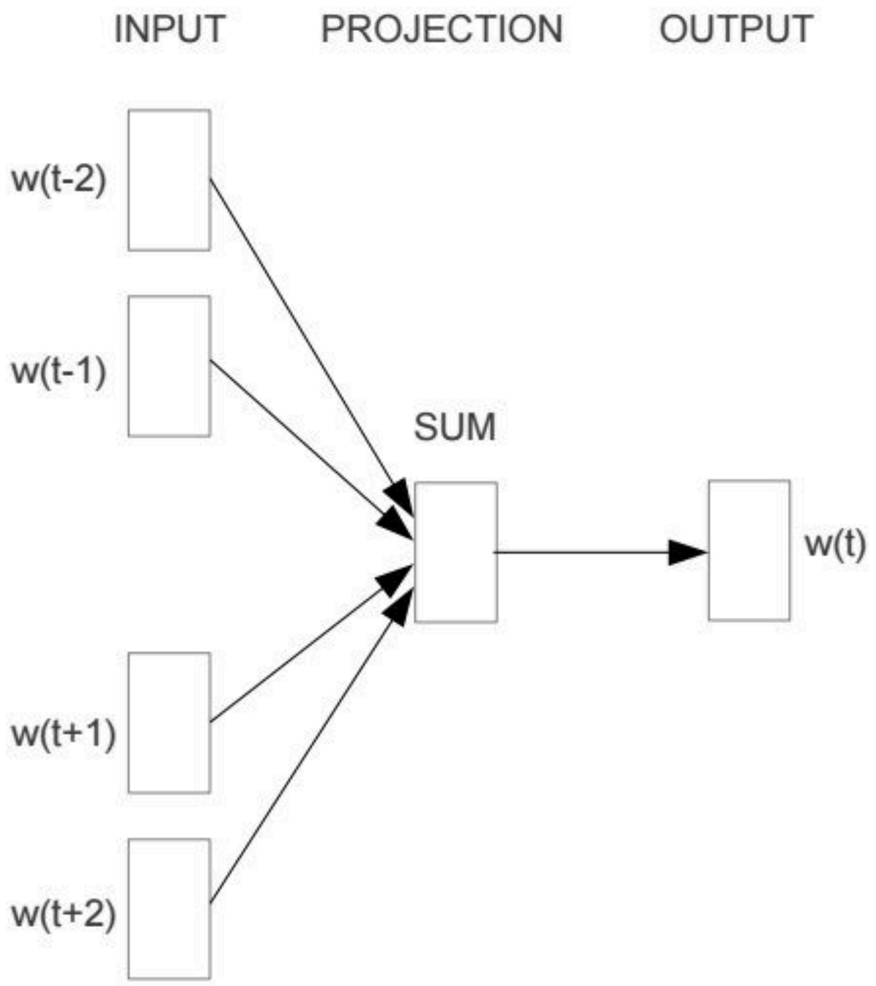


1. Treat the target word and a neighboring context word as positive examples.
2. Randomly sample other words in the lexicon to get negative samples.
3. Use logistic regression to train a classifier to distinguish those two cases.
4. Use the learned weights as the embeddings.

# Continuous Bag of Words (CBOW)



Skip-gram



Continuous Bag of Words (CBOW)

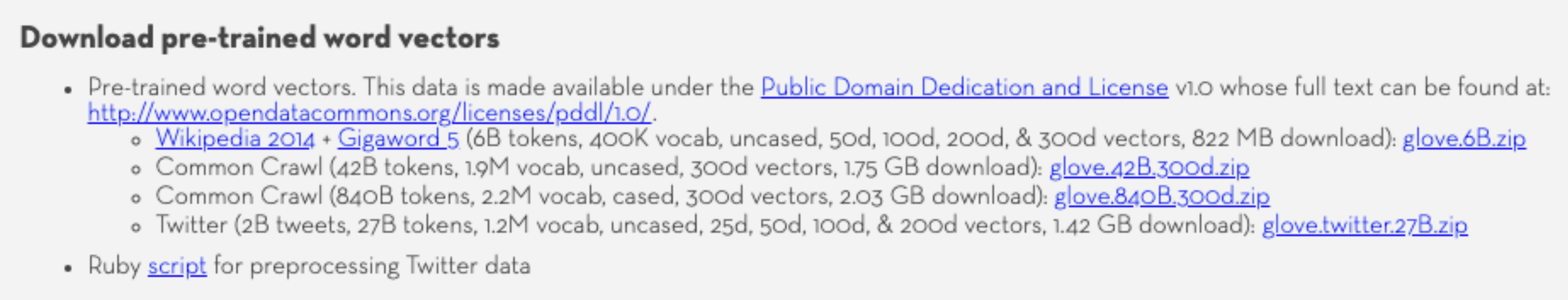
$$L(\theta) = \prod_{t=1}^{T} P\left(w_t \mid \{w_{t+j}\}, -m \leq j \leq m, j \neq 0\right)$$

$$\bar{\mathbf{v}}_t = \frac{1}{2m} \sum_{-m \leq j \leq m, j \neq 0} \mathbf{v}_{t+j}$$

$$P(w_t \mid \{w_{t+j}\}) = \frac{\exp(\mathbf{u}_{w_t} \cdot \bar{\mathbf{v}}_t)}{\sum_{k \in V} \exp(\mathbf{u}_k \cdot \bar{\mathbf{v}}_t)}$$

# Trained word embeddings available

- word2vec: https://code.google.com/archive/p/word2vec/
- GloVe: https://nlp.stanford.edu/projects/glove/
- FastText: https://fasttext.cc/



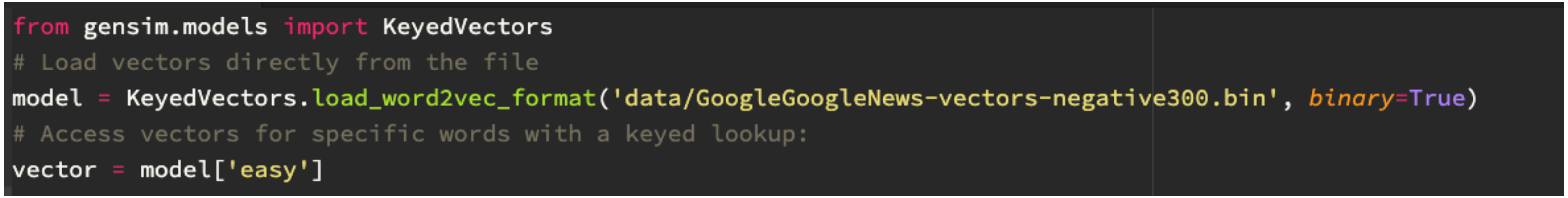
### Download pre-trained word vectors

- Pre-trained word vectors. This data is made available under the Public Domain Dedication and License v1.0 whose full text can be found at: http://www.opendatacommons.org/licenses/pddl/1.0/.
  - Wikipedia 2014 + Gigaword 5 (6B tokens, 400K vocab, uncased, 50d, 100d, 200d, & 300d vectors, 822 MB download): glove.6B.zip
  - Common Crawl (42B tokens, 1.9M vocab, uncased, 300d vectors, 1.75 GB download): glove.42B.300d.zip
  - Common Crawl (840B tokens, 2.2M vocab, cased, 300d vectors, 2.03 GB download): glove.840B.300d.zip
  - Twitter (2B tweets, 27B tokens, 1.2M vocab, uncased, 25d, 50d, 100d, & 200d vectors, 1.42 GB download): glove.twitter.27B.zip
- Ruby script for preprocessing Twitter data

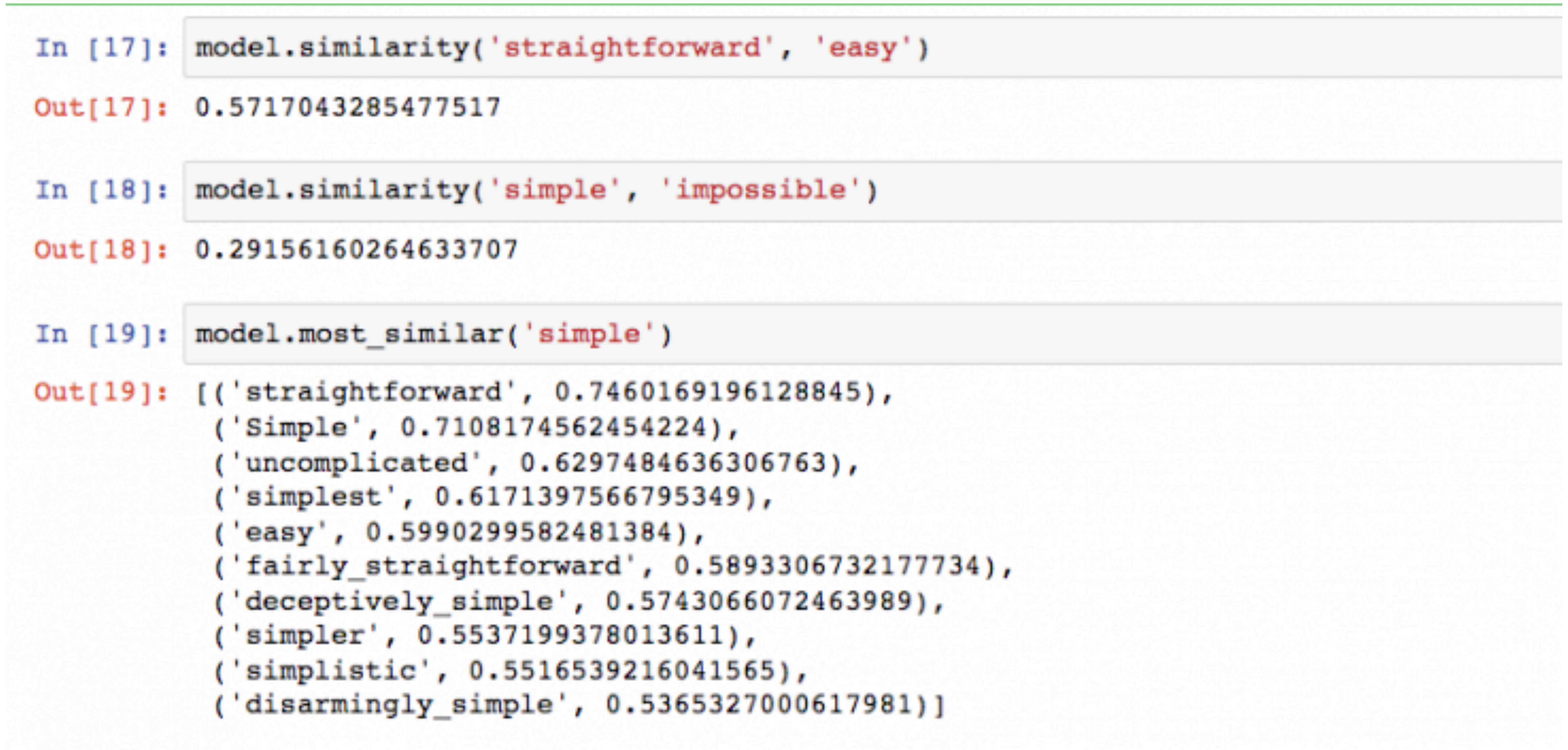
Differ in algorithms, text corpora, dimensions, cased/uncased…

Applied to many other languages

# Easy to use!



```python
from gensim.models import KeyedVectors
# Load vectors directly from the file
model = KeyedVectors.load_word2vec_format('data/GoogleGoogleNews-vectors-negative300.bin', binary=True)
# Access vectors for specific words with a keyed lookup:
vector = model['easy']
```



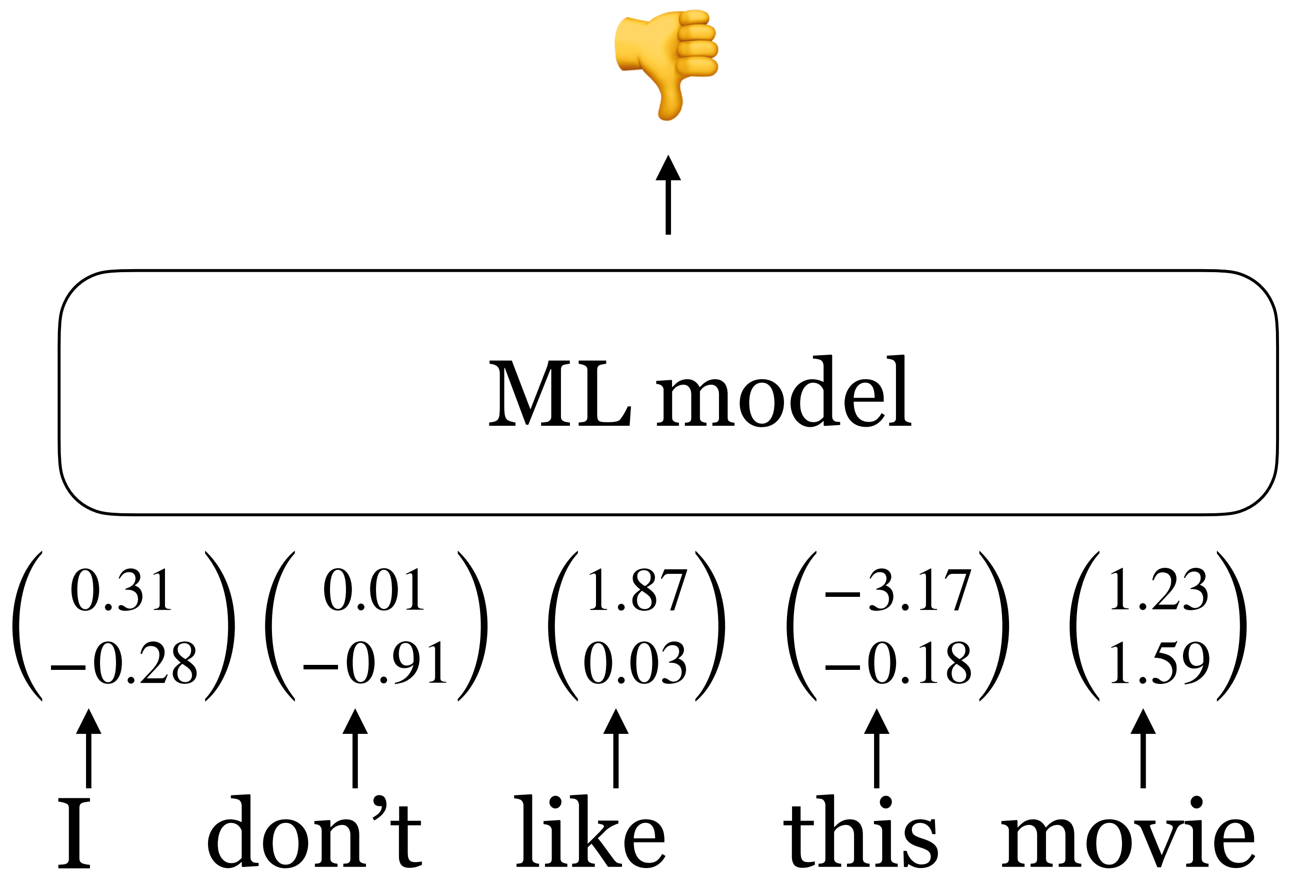
```
In [17]: model.similarity('straightforward', 'easy')

Out[17]: 0.5717043285477517

In [18]: model.similarity('simple', 'impossible')

Out[18]: 0.29156160264633707

In [19]: model.most_similar('simple')

Out[19]: [('straightforward', 0.7460169196128845),
          ('Simple', 0.7108174562454224),
          ('uncomplicated', 0.6297484636306763),
          ('simplest', 0.6171397566795349),
          ('easy', 0.5990299582481384),
          ('fairly_straightforward', 0.5893306732177734),
          ('deceptively_simple', 0.5743066072463989),
          ('simpler', 0.5537199378013611),
          ('simplistic', 0.5516539216041565),
          ('disarmingly_simple', 0.5365327000617981)]
```

# Evaluating word embeddings

# Extrinsic vs intrinsic evaluation

**Extrinsic evaluation**

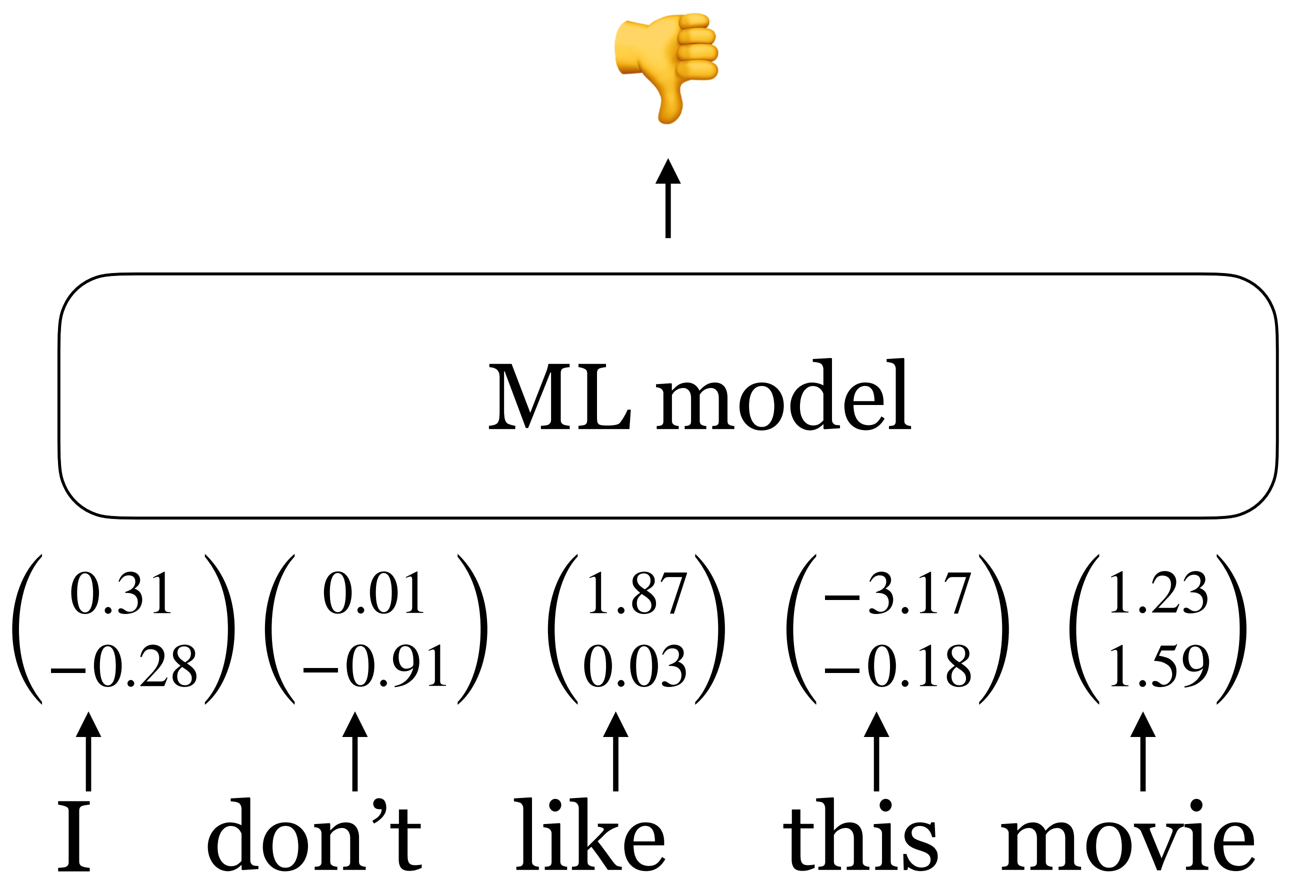
- Let’s plug these word embeddings into a real NLP system and see whether this improves performance
- Could take a long time but still the most important evaluation metric



**Intrinsic evaluation**

- Evaluate on a specific/intermediate subtask
- Fast to compute
- Not clear if it really helps downstream tasks

# Extrinsic evaluation



A straightforward solution: given an input sentence $x_1, x_2, \ldots, x_n$

Instead of using a bag-of-words model, we can compute $vec(x) = \mathbf{e}(x_1) + \mathbf{e}(x_2) + \ldots + \mathbf{e}(x_n)$

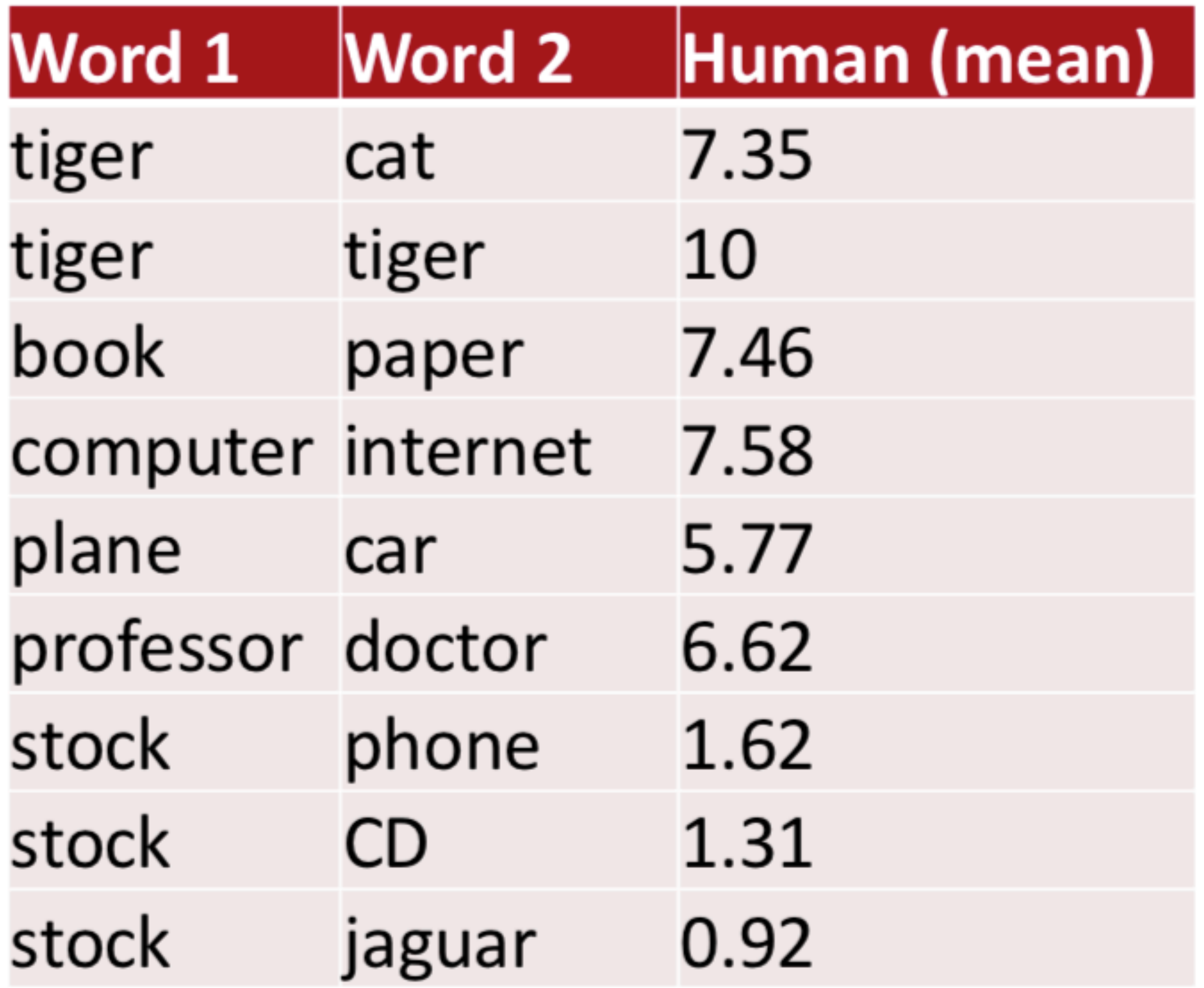
And then train a logistic regression classifier on $vec(x)$ as we did before!

There are much better ways to do this e.g., take word embeddings as input of neural networks

# Intrinsic evaluation: word similarity

## Word similarity

Example dataset: wordsim-353
353 pairs of words with human judgement
http://www.cs.technion.ac.il/~gabr/resources/data/wordsim353/



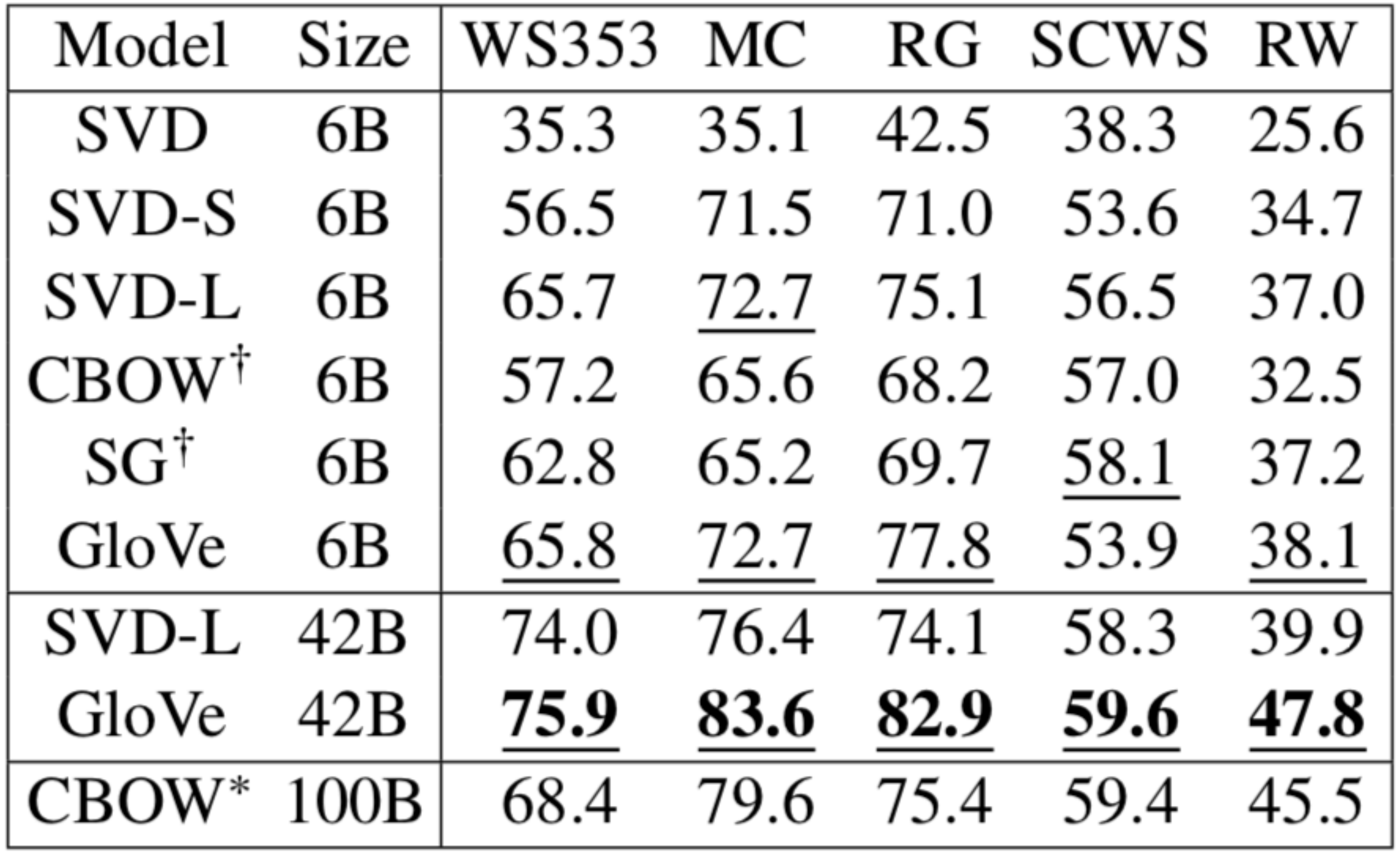
| Word 1 | Word 2 | Human (mean) |
|---|---|---|
| tiger | cat | 7.35 |
| tiger | tiger | 10 |
| book | paper | 7.46 |
| computer | internet | 7.58 |
| plane | car | 5.77 |
| professor | doctor | 6.62 |
| stock | phone | 1.62 |
| stock | CD | 1.31 |
| stock | jaguar | 0.92 |

Cosine similarity:

$$\cos(\boldsymbol{u}_i, \boldsymbol{u}_j) = \frac{\boldsymbol{u}_i \cdot \boldsymbol{u}_j}{||\boldsymbol{u}_i||_2 \times ||\boldsymbol{u}_j||_2}.$$

Metric: Spearman rank correlation

# Intrinsic evaluation: word similarity



| Model | Size | WS353 | MC | RG | SCWS | RW |
|---|---|---|---|---|---|---|
| SVD | 6B | 35.3 | 35.1 | 42.5 | 38.3 | 25.6 |
| SVD-S | 6B | 56.5 | 71.5 | 71.0 | 53.6 | 34.7 |
| SVD-L | 6B | 65.7 | 72.7 | 75.1 | 56.5 | 37.0 |
| CBOW† | 6B | 57.2 | 65.6 | 68.2 | 57.0 | 32.5 |
| SG† | 6B | 62.8 | 65.2 | 69.7 | 58.1 | 37.2 |
| GloVe | 6B | 65.8 | 72.7 | 77.8 | 53.9 | 38.1 |
| SVD-L | 42B | 74.0 | 76.4 | 74.1 | 58.3 | 39.9 |
| GloVe | 42B | **75.9** | **83.6** | **82.9** | **59.6** | **47.8** |
| CBOW* | 100B | 68.4 | 79.6 | 75.4 | 59.4 | 45.5 |

SG: Skip-gram

# Intrinsic evaluation: word analogy

Word analogy test: $a : a* :: b : b*$

$$b^* = \arg\max_{w \in V} \cos(e(w), e(a^*) - e(a) + e(b))$$

semantic

Chicago:Illinois≈Philadelphia: ?

syntactic

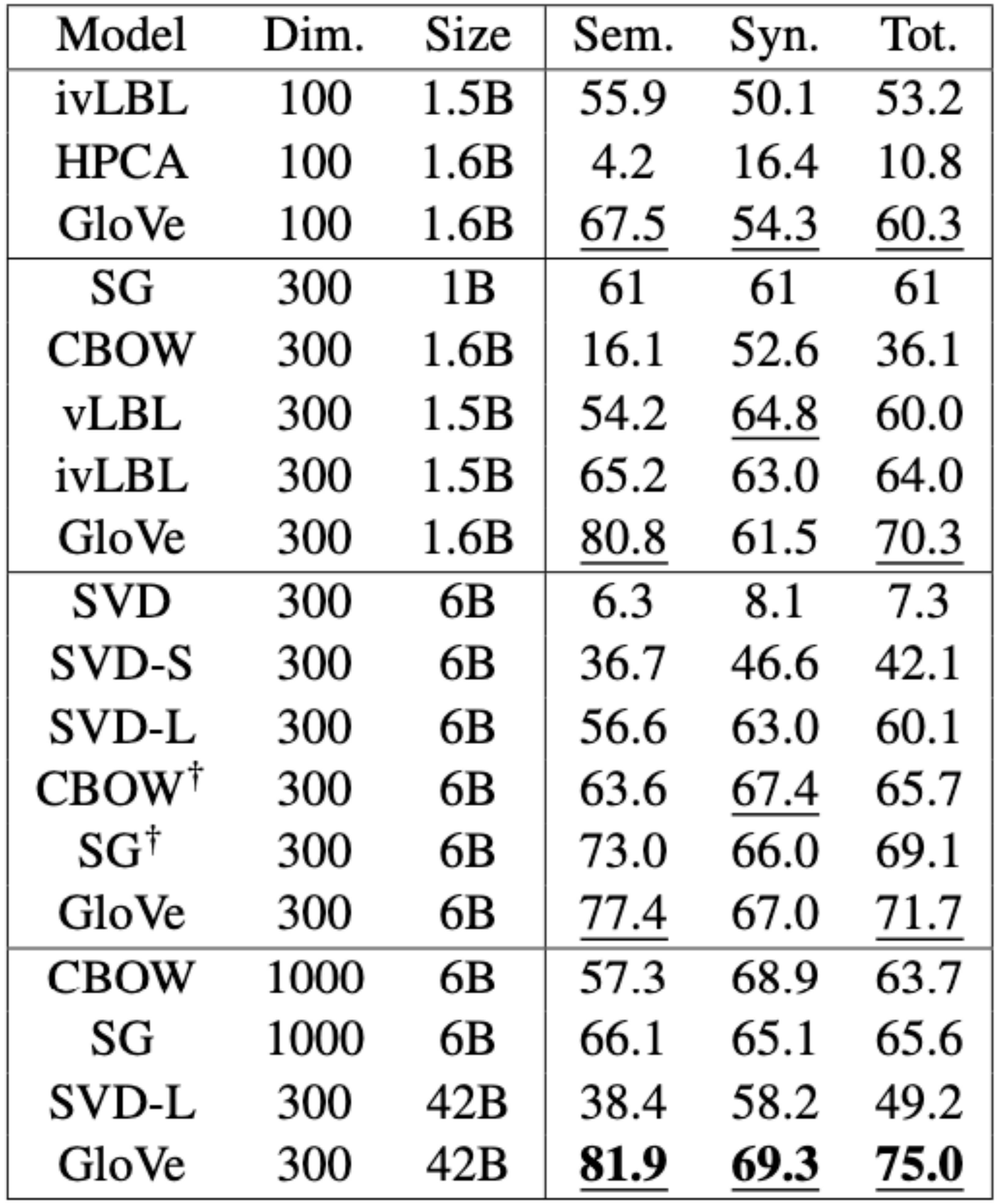
bad:worst ≈ cool: ?

More examples at

http://download.tensorflow.org/data/questions-words.txt

Metric: accuracy

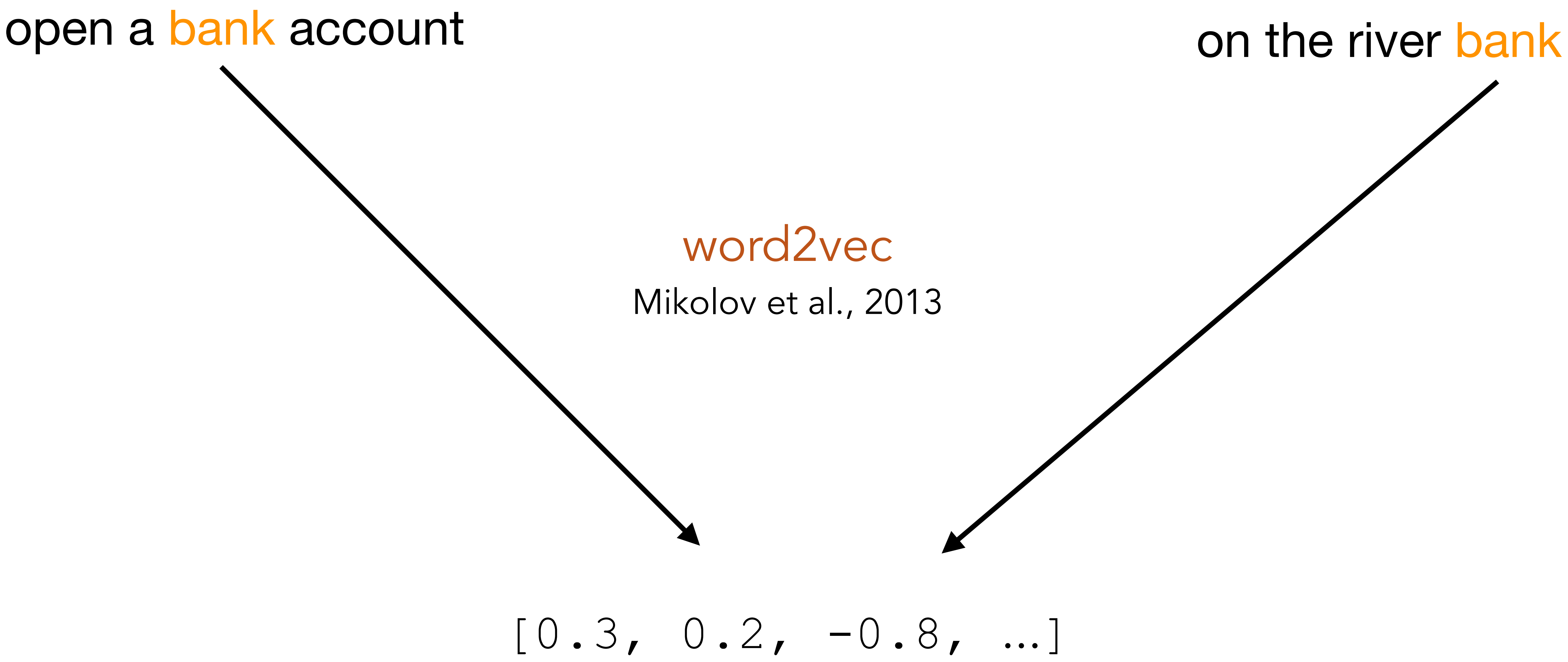
# Intrinsic evaluation: word analogy



| Model | Dim. | Size | Sem. | Syn. | Tot. |
|---|---|---|---|---|---|
| ivLBL | 100 | 1.5B | 55.9 | 50.1 | 53.2 |
| HPCA | 100 | 1.6B | 4.2 | 16.4 | 10.8 |
| GloVe | 100 | 1.6B | 67.5 | 54.3 | 60.3 |
| SG | 300 | 1B | 61 | 61 | 61 |
| CBOW | 300 | 1.6B | 16.1 | 52.6 | 36.1 |
| vLBL | 300 | 1.5B | 54.2 | 64.8 | 60.0 |
| ivLBL | 300 | 1.5B | 65.2 | 63.0 | 64.0 |
| GloVe | 300 | 1.6B | 80.8 | 61.5 | 70.3 |
| SVD | 300 | 6B | 6.3 | 8.1 | 7.3 |
| SVD-S | 300 | 6B | 36.7 | 46.6 | 42.1 |
| SVD-L | 300 | 6B | 56.6 | 63.0 | 60.1 |
| CBOW† | 300 | 6B | 63.6 | 67.4 | 65.7 |
| SG† | 300 | 6B | 73.0 | 66.0 | 69.1 |
| GloVe | 300 | 6B | 77.4 | 67.0 | 71.7 |
| CBOW | 1000 | 6B | 57.3 | 68.9 | 63.7 |
| SG | 1000 | 6B | 66.1 | 65.1 | 65.6 |
| SVD-L | 300 | 42B | 38.4 | 58.2 | 49.2 |
| GloVe | 300 | 42B | **81.9** | **69.3** | **75.0** |

# Static embeddings: word2vec

word2vec: context-free word embeddings



open a bank account

on the river bank

word2vec

Mikolov et al., 2013

```
[0.3, 0.2, -0.8, …]
```

# Contextual embeddings: BERT, GPT…

BERT: contextual word embeddings

open a bank account

[0.9, -0.2, 1.6, …]

BERT

Devlin et al., 2018

on the river bank

[-1.9, -0.4, 0.1, …]

We will introduce BERT/GPT soon in this class!