

COMP 3361 Natural Language Processing

Lecture 11: Attention and Transformers

Many materials from CSE447@UW (Liwei Jiang), COS 484@Princeton, and CS224n@Stanford with special thanks!

Spring 2025

Announcements

- Assignment 2 is out, due on April 1.
 - Written questions resemble the types of questions that could appear on your final exam.

Transformers

Attention Is All You Need

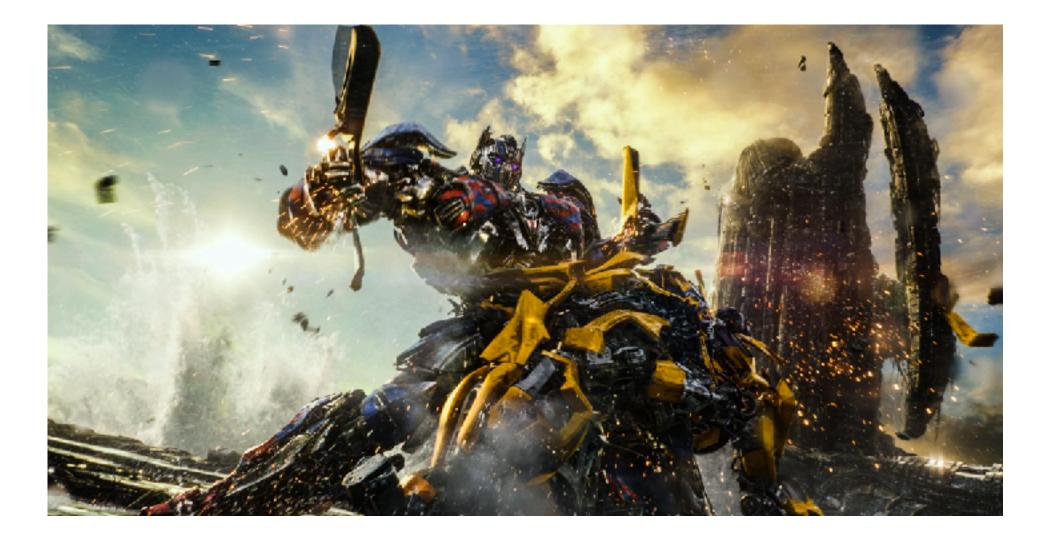
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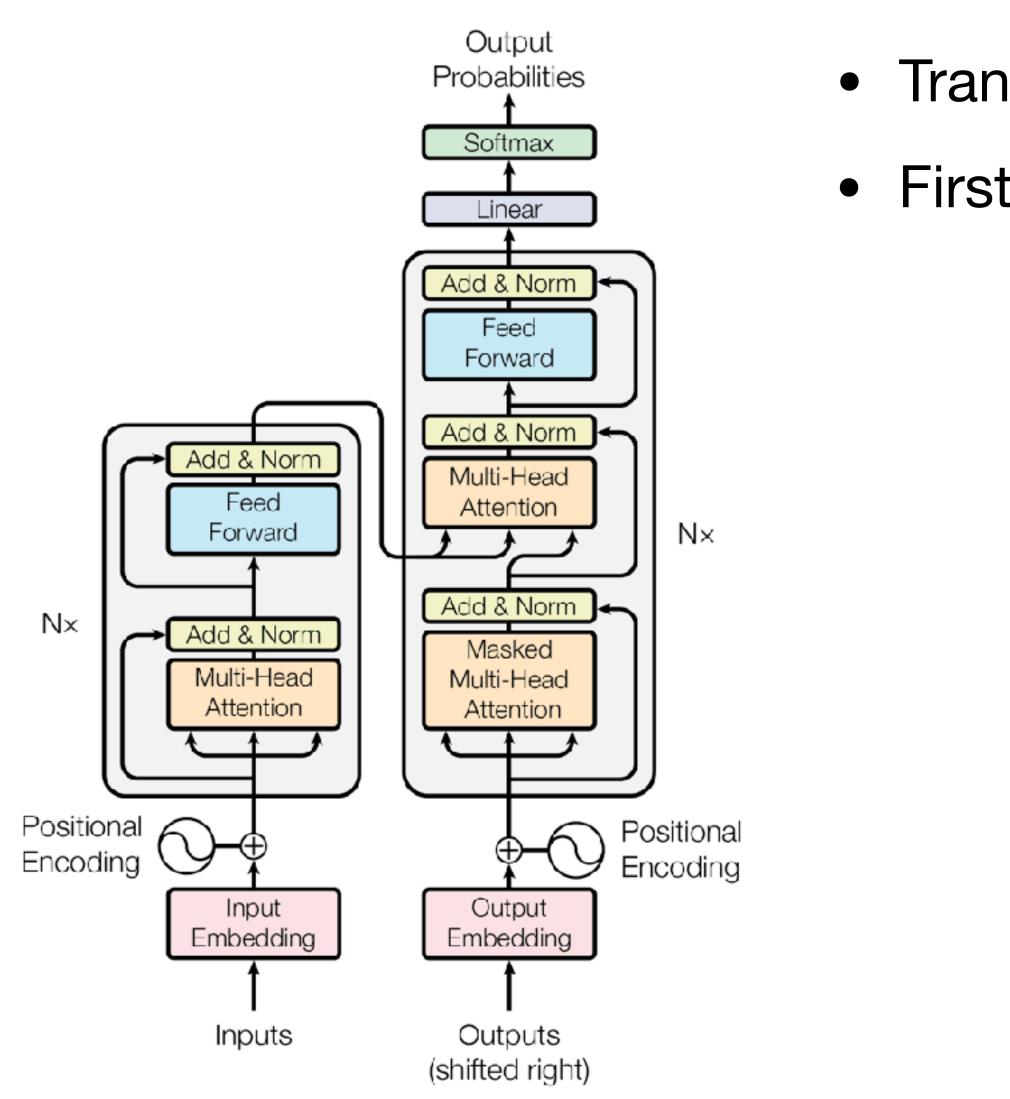
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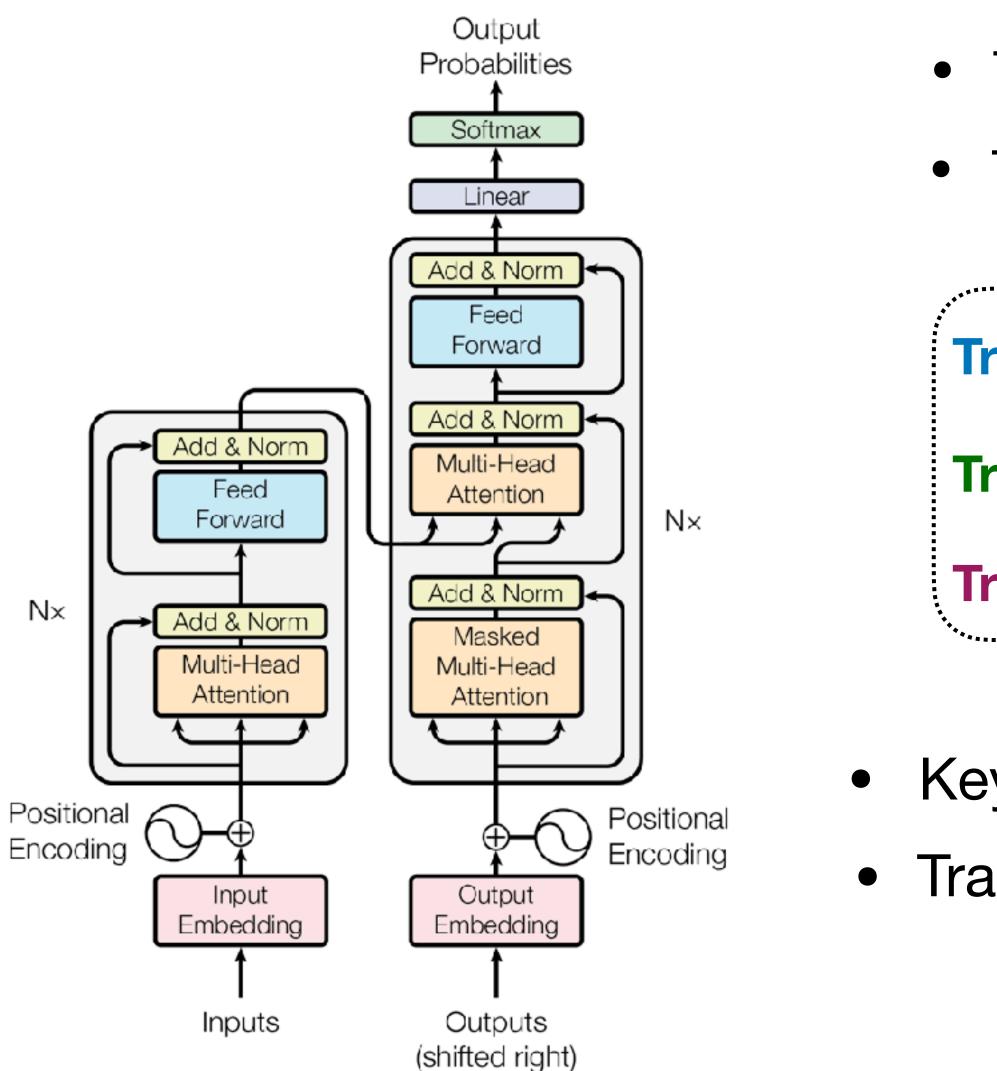
(Vaswani et al., 2017)





Transformer encoder-decoder

- Transformer encoder + Transformer decoder
- First designed and experimented on NMT



Transformer encoder-decoder

- Transformer encoder = a stack of **encoder layers**
- Transformer decoder = a stack of **decoder layers**

Transformer encoder: BERT, RoBERTa, ELECTRA **Transformer decoder**: GPT-3, ChatGPT, Palm **Transformer encoder-decoder:** T5, BART

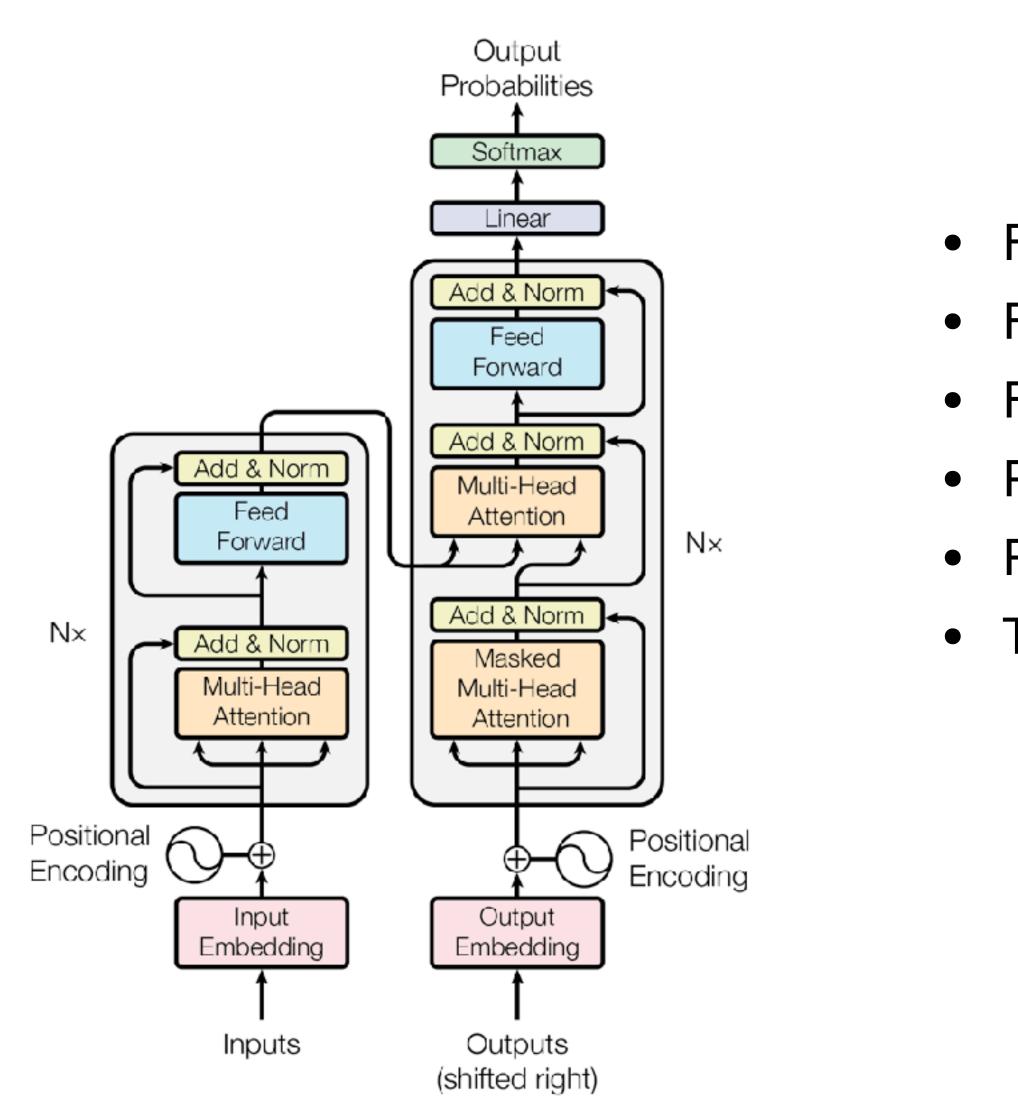
- Key innovation: multi-head, self-attention
- Transformers don't have any recurrence structures!

$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t) \in \mathbb{R}^h$$





Transformers: roadmap



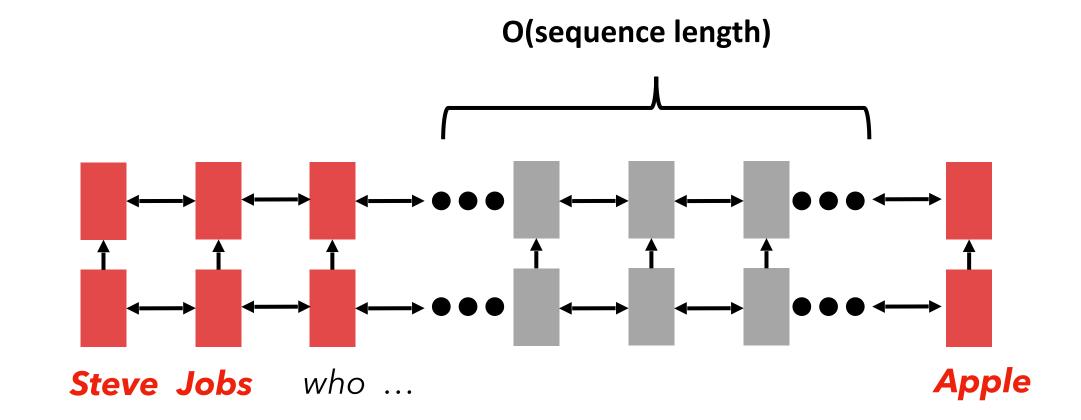
- From attention to self-attention
- From self-attention to multi-head self-attention
- Feedforward layers
 - Positional encoding
- Residual connections + layer normalization
 - Transformer encoder vs Transformer decoder

Issues with RNNs: Linear Interaction Distance

- RNNs are unrolled **left-to-right**.
 - Linear locality is a useful heuristic: nearby words often affect each other's meaning!
- However, there's the **vanishing gradient** problem for long sequences.
 - The gradients that are used to update the network become extremely small or "vanish" as they are backpropogated from the output layers to the earlier layers.



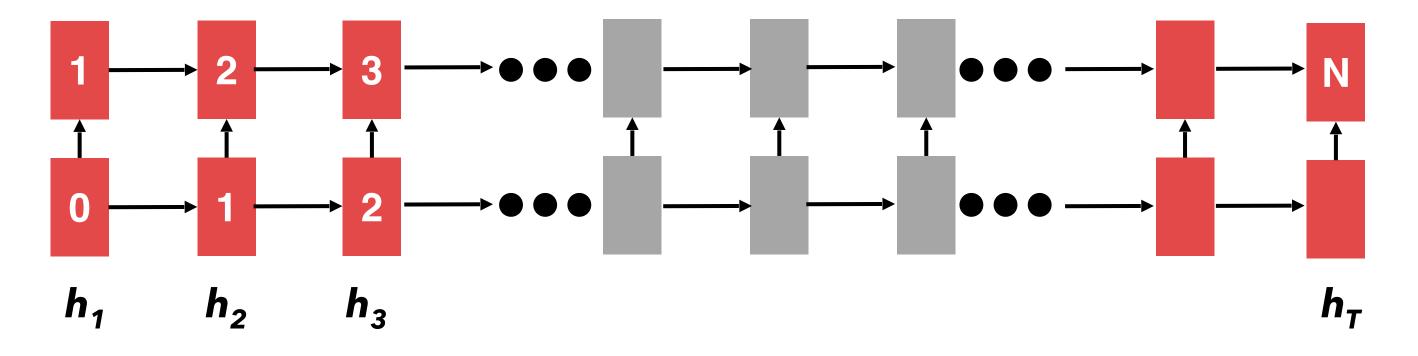
Steve



Jobs

Issues with RNNs: Lack of Parallelizability

- - GPUs can perform many independent computations (like addition) at once!
 - But future RNN hidden states can't be computed in full before past RNN hidden states have been computed.
 - Training and inference are slow; inhibits on very large datasets!

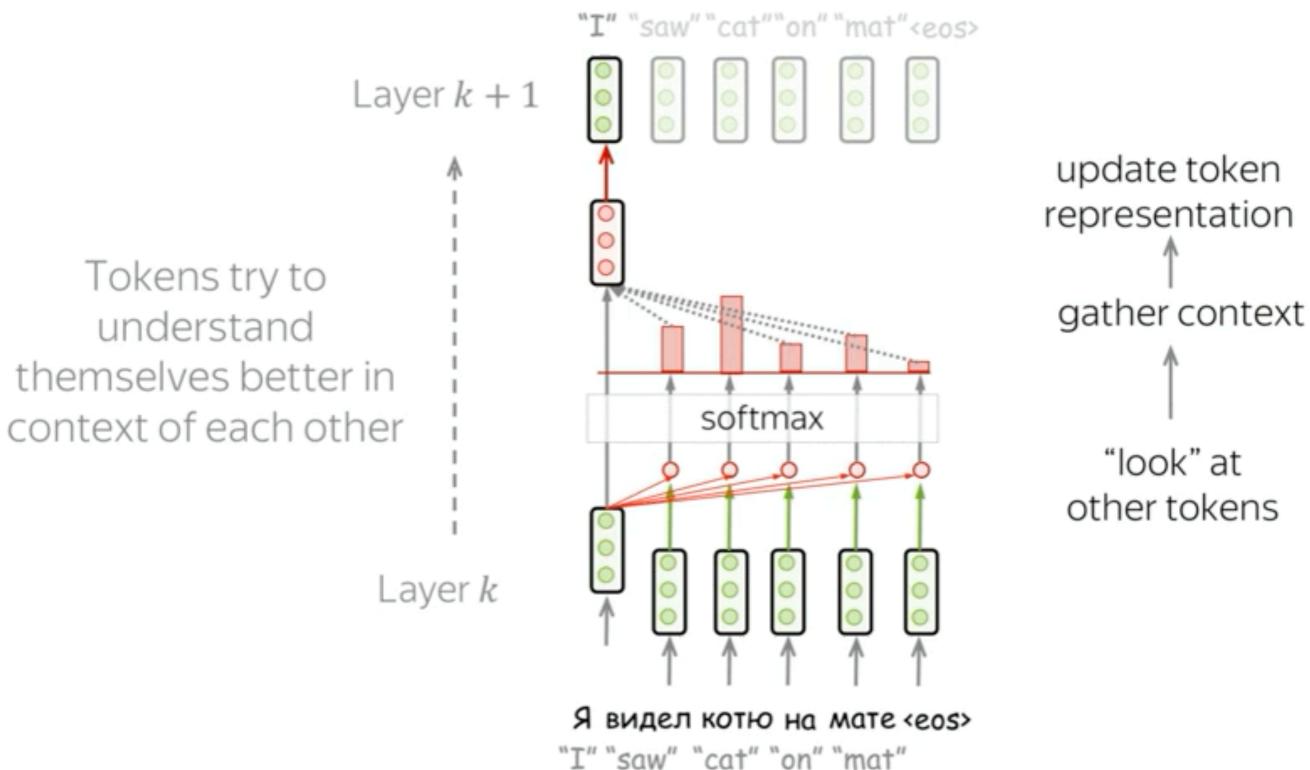


• Forward and backward passes have **O(sequence length) unparallelizable** operations

Numbers indicate min # of steps before a state can be computed

The New De Facto Method: Attention

Instead of deciding the next token solely based on the previously seen tokens, each token will "look at" all input tokens at the same to decide which ones are most important to decide the next token.

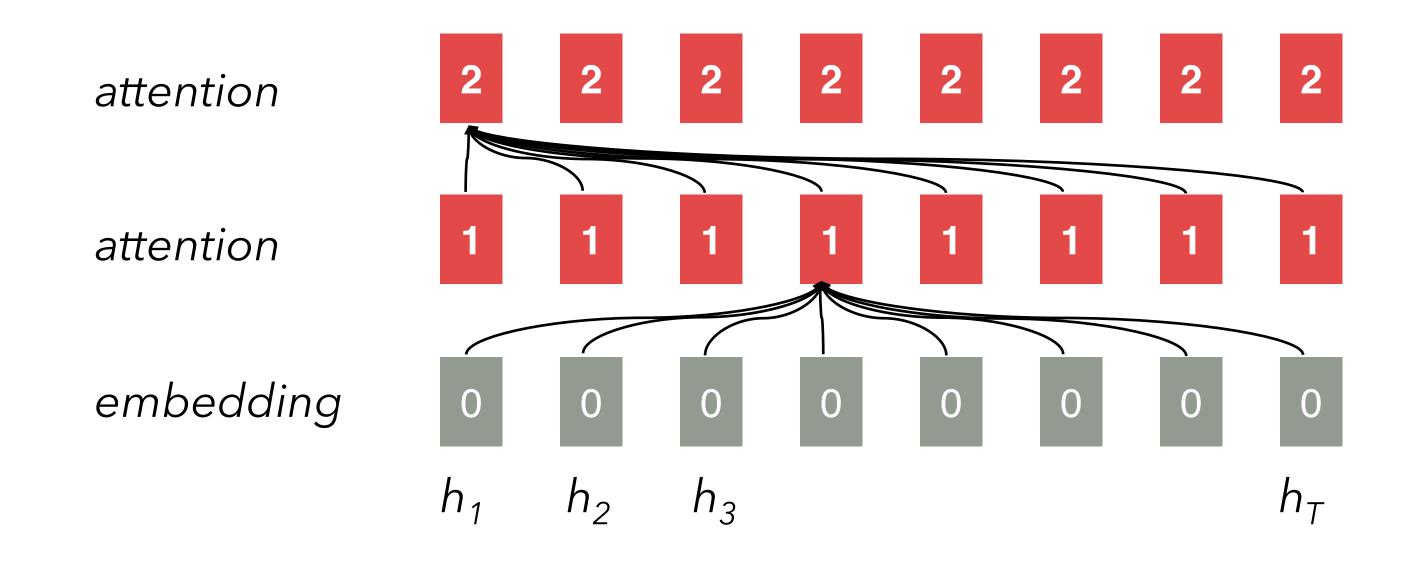


In practice, the actions of all tokens are done in parallel!



Building the Intuition of Attention

- information from **a set of values**.
 - Today we look at attention within a single sequence.
- Number of unparallelizable operations does **NOT** increase with sequence length.
- Maximum interaction distance: O(1), since all tokens interact at every layer!



• Attention treats each token's representation as a query to access and incorporate

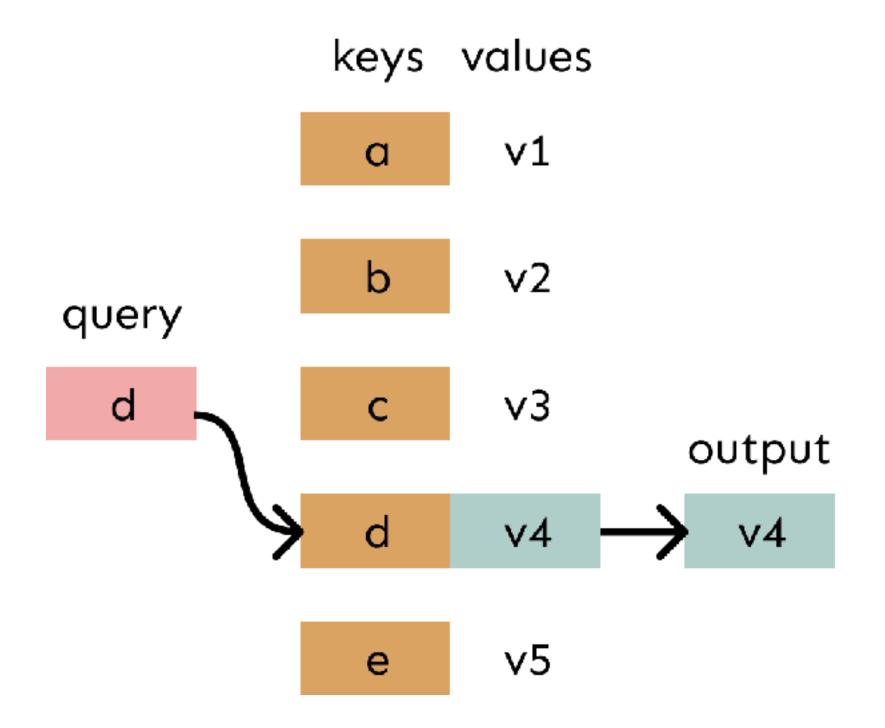
All tokens attend to all tokens in previous layer; most arrows here are omitted



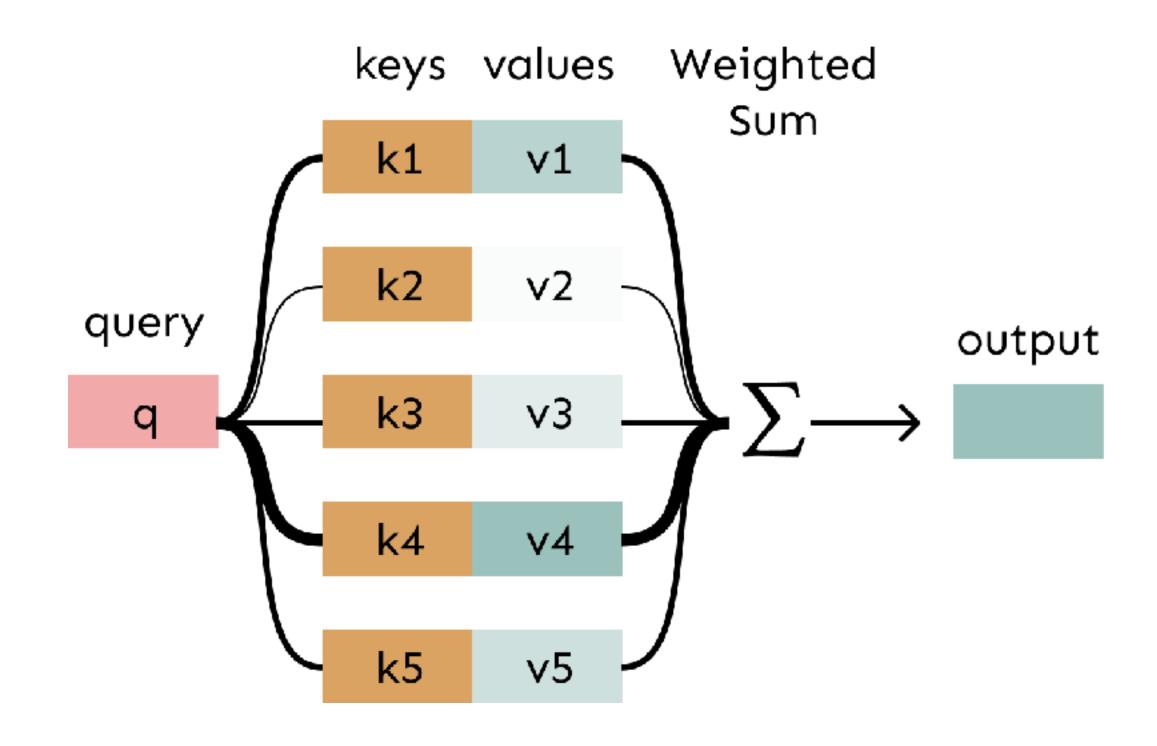
Attention as a soft, averaging lookup table

We can think of attention as performing fuzzy lookup in a key-value store.

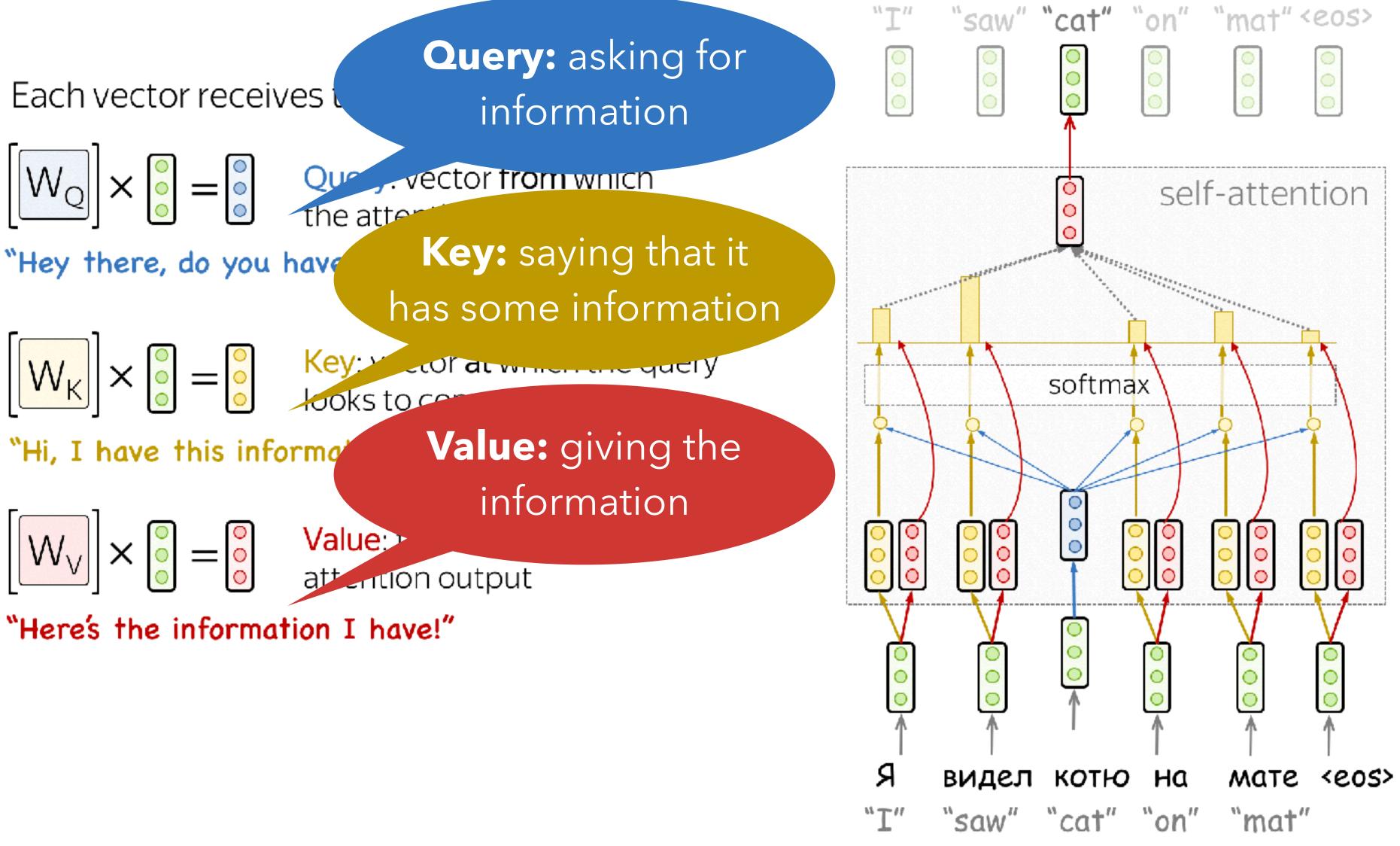
In a **lookup table**, we have a table of **keys** that map to **values**. The **query** matches one of the keys, returning its value.



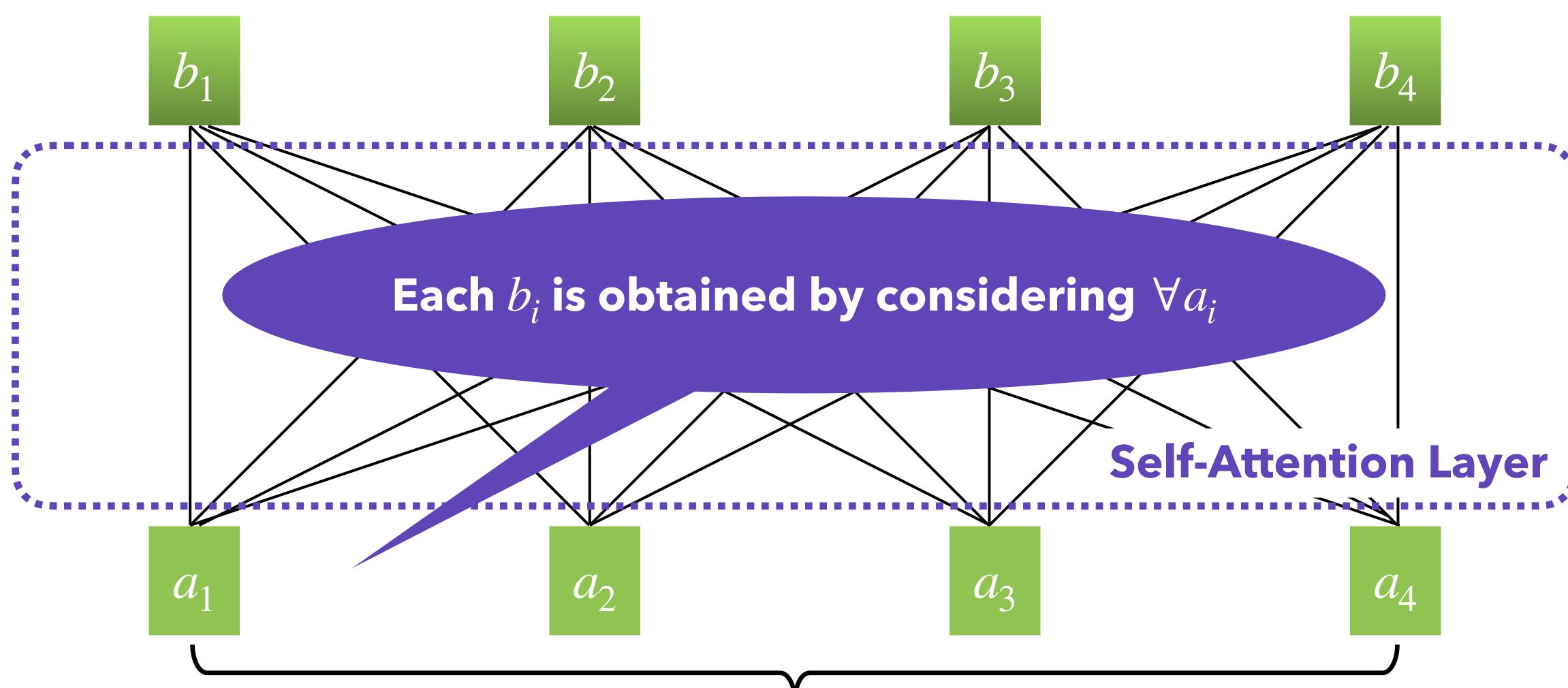
In **attention**, the **query** matches all **keys** *softly*, to a weight between 0 and 1. The keys' **values** are multiplied by the weights and summed.



Self-Attention: Basic Concepts



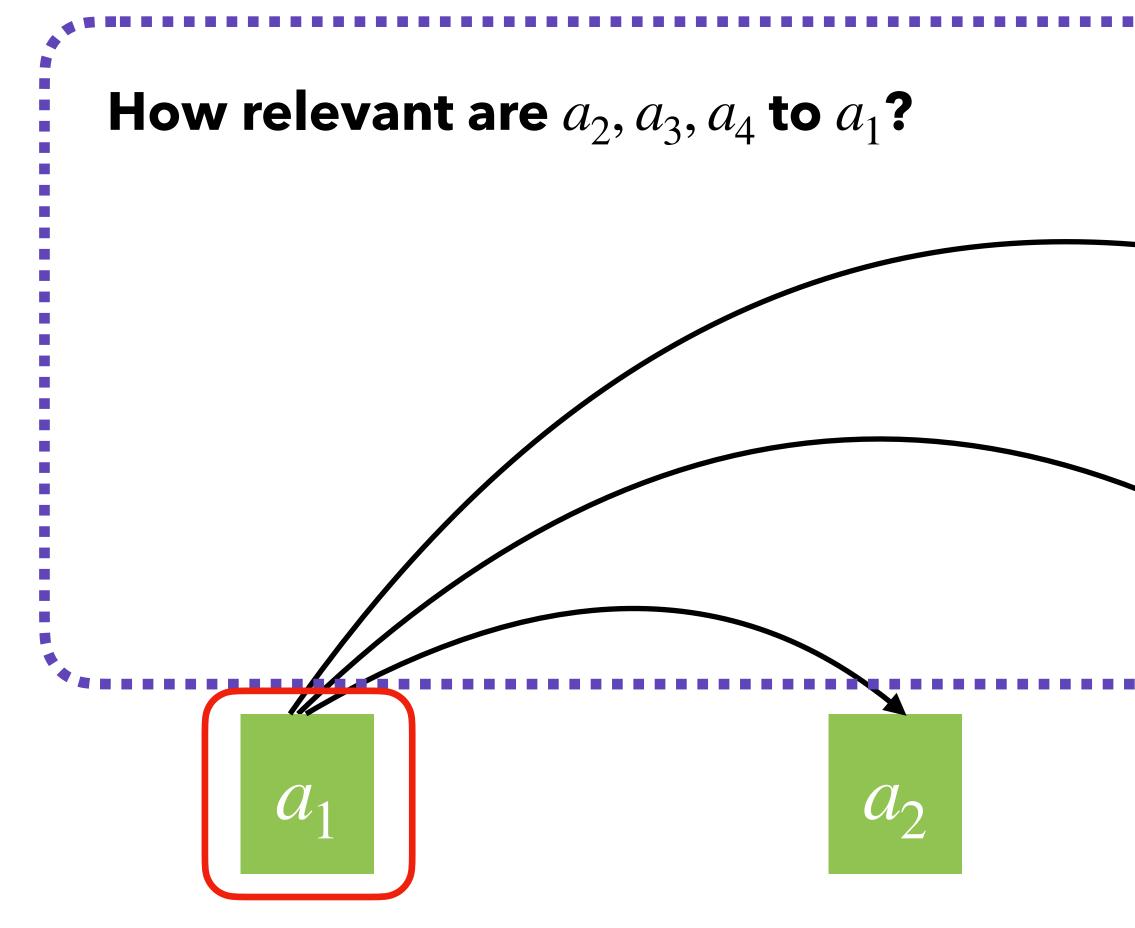
[Lena Viota Blog]



Can be either input or a hidden layer







We denote the level

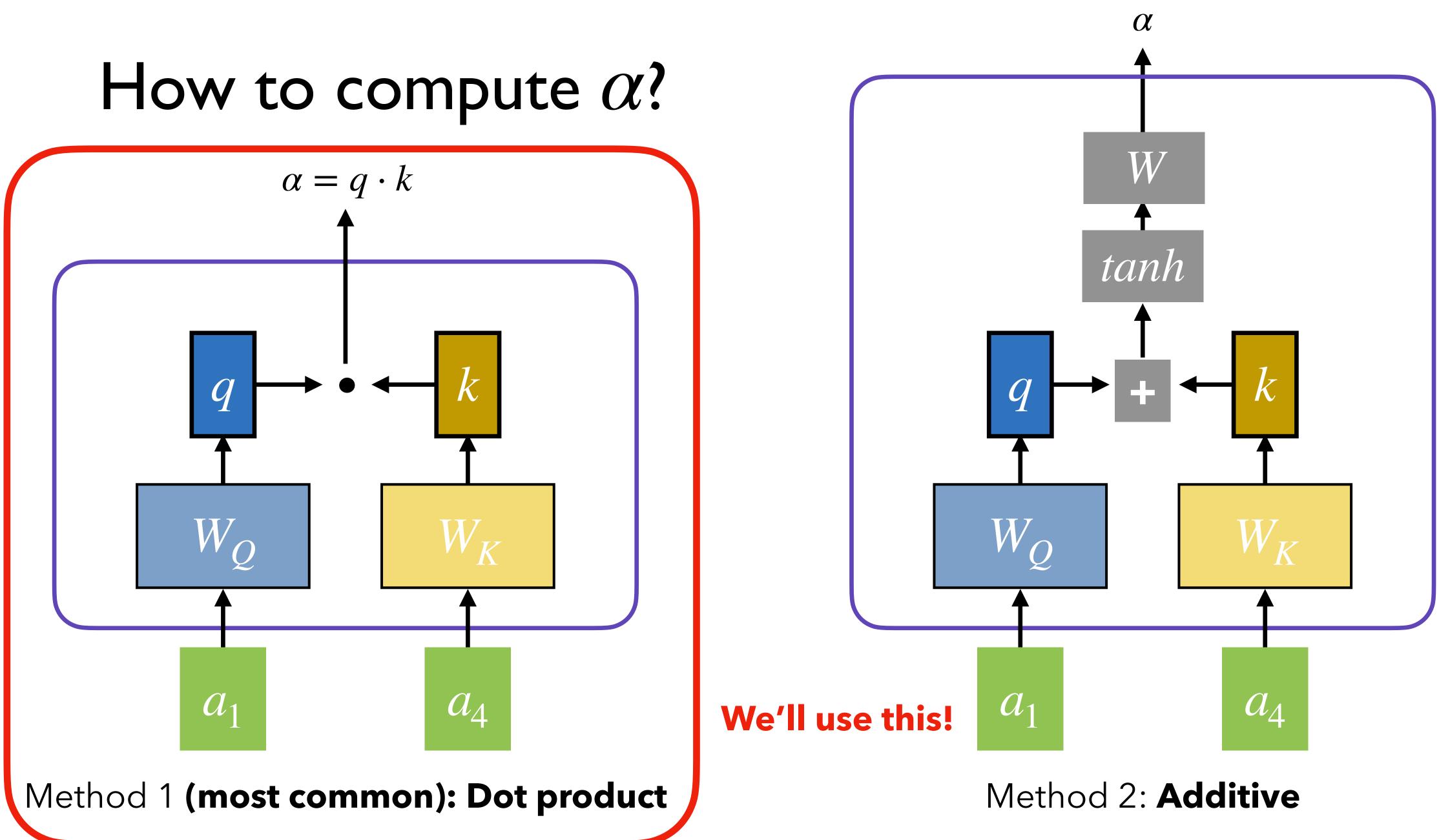
of relevance as α

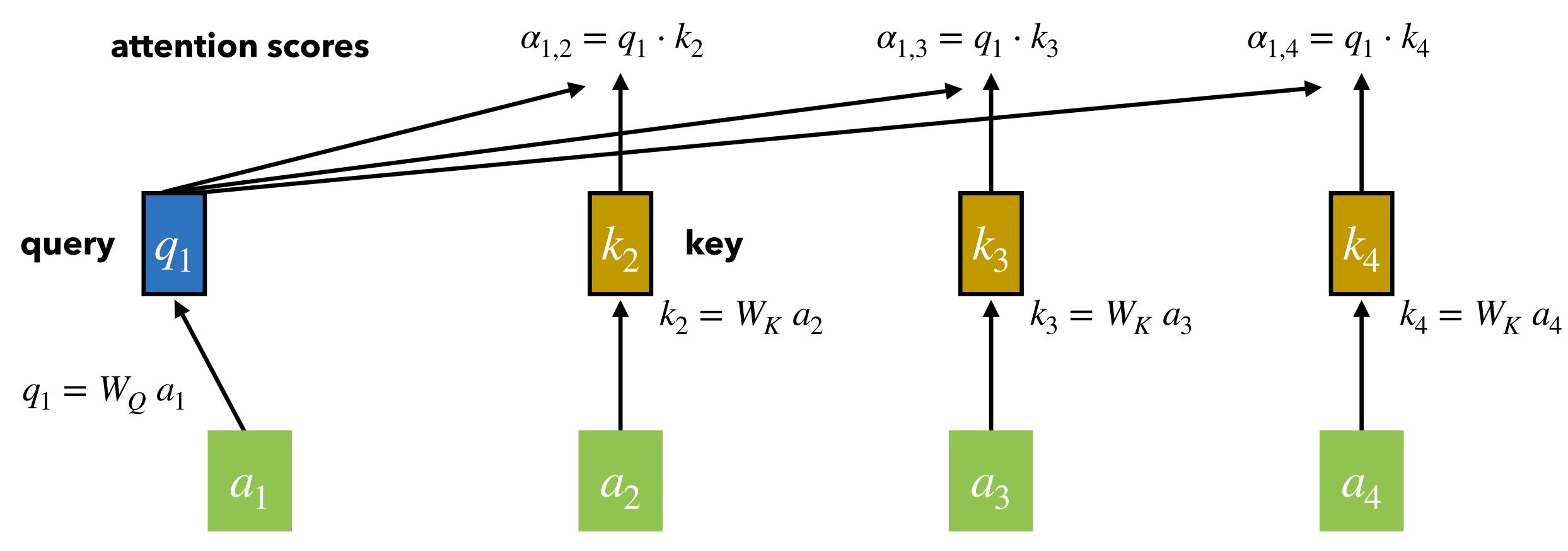




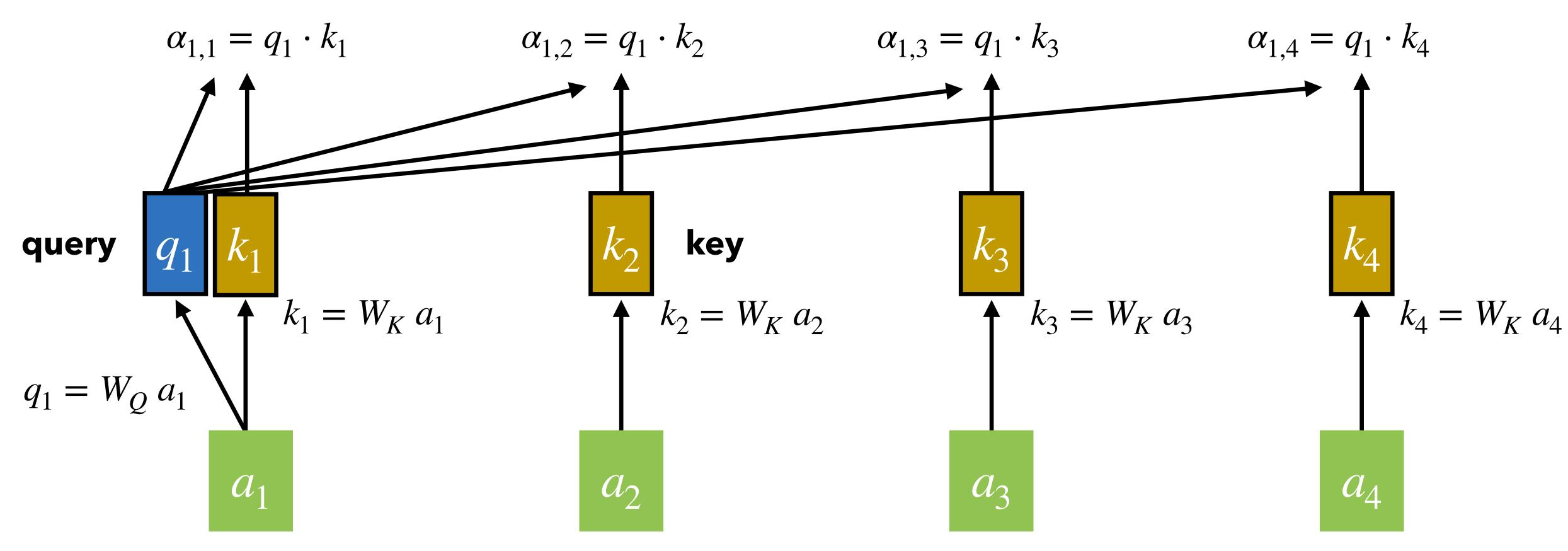
X

 \mathcal{A}_{Δ}

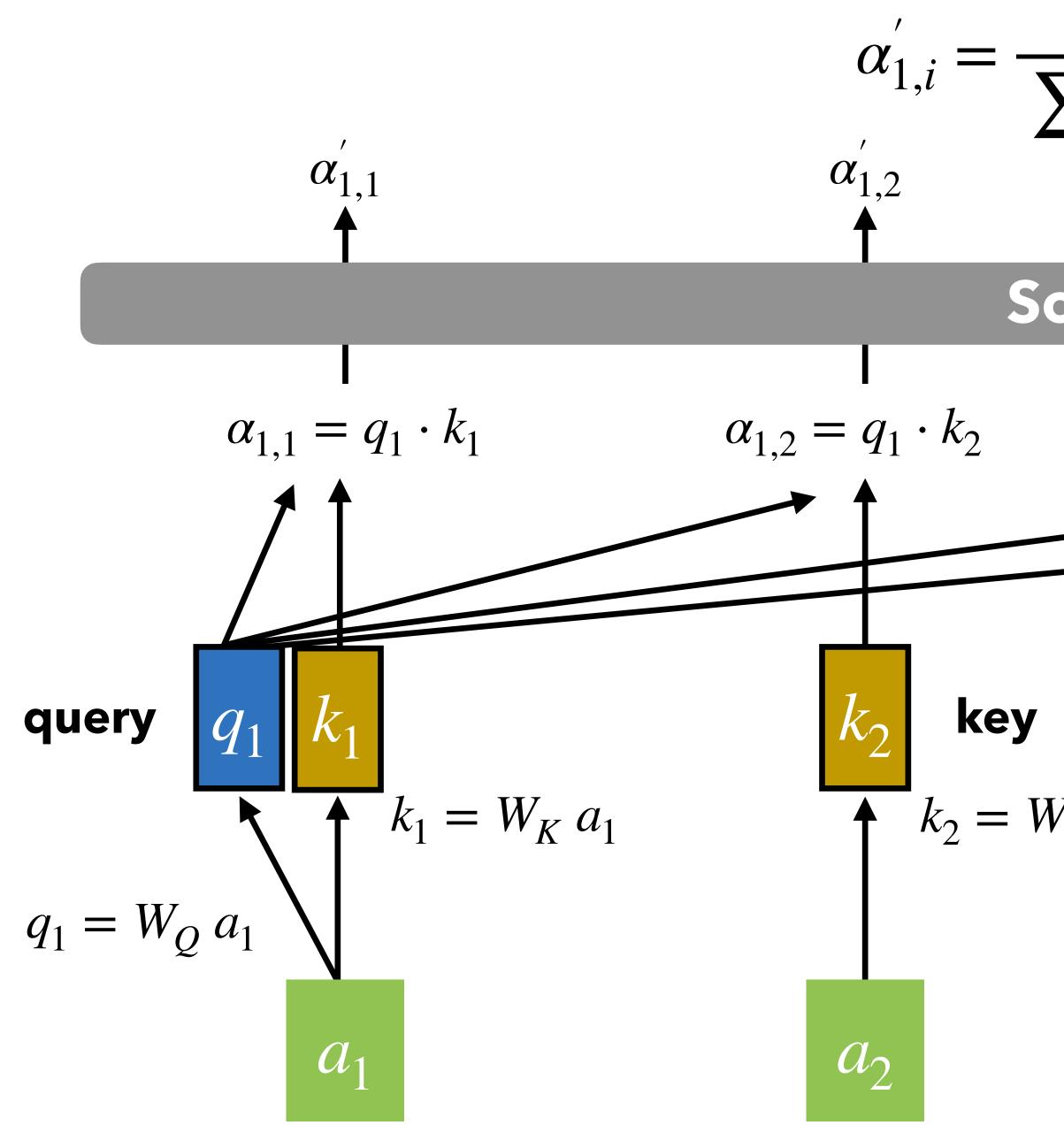










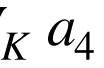


$$\frac{e^{\alpha_{1,i}}}{\sum_{j} e^{\alpha_{1,j}}}$$

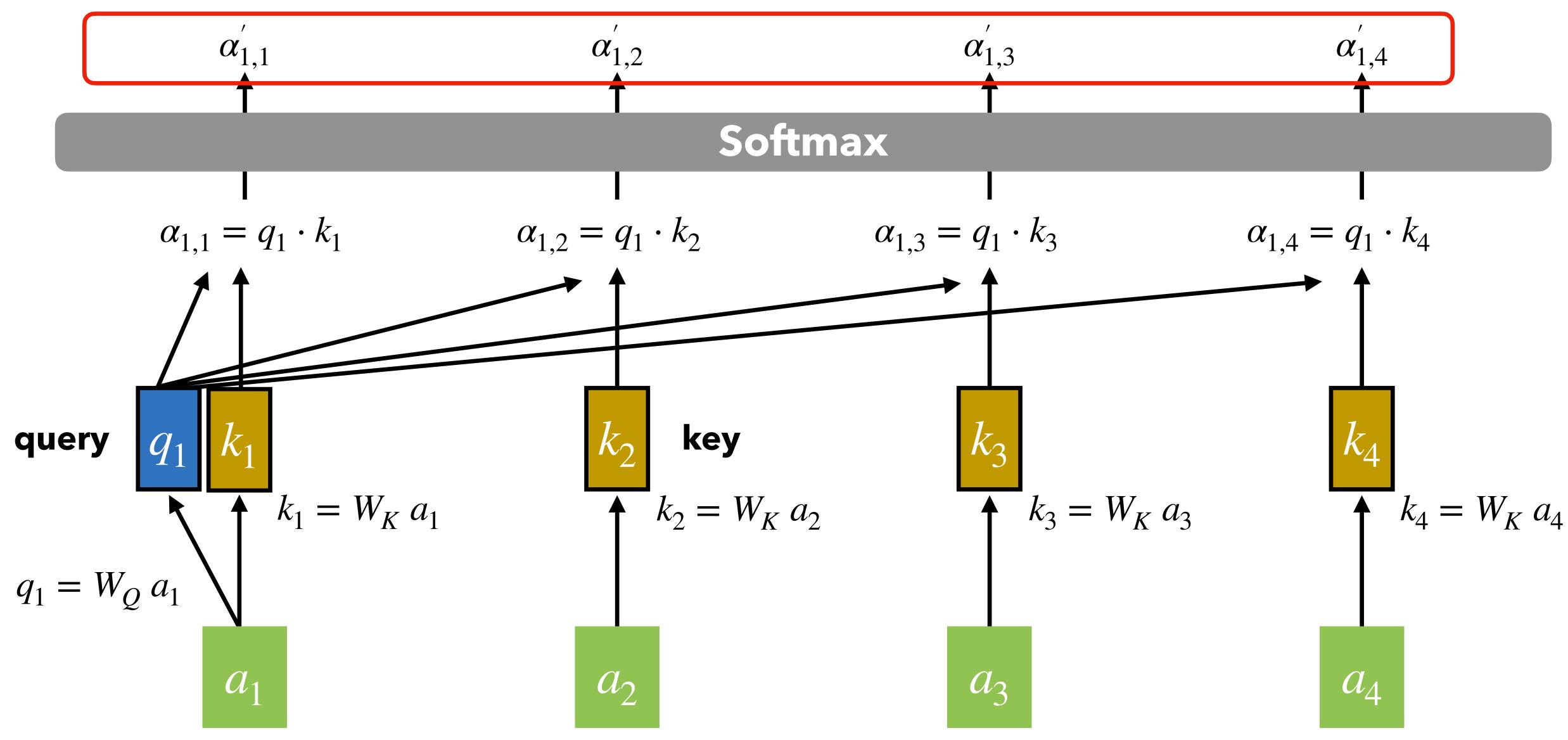
$$\alpha'_{1,3} \qquad \alpha'_{1,4} \qquad \alpha'_{1,4}$$
oftmax
$$\alpha_{1,3} = q_1 \cdot k_3 \qquad \alpha_{1,4} = q_1 \cdot k_4$$

$$k_3 \qquad k_3 = W_K a_3 \qquad k_4 = W_I$$

$$a_3 \qquad a_4$$



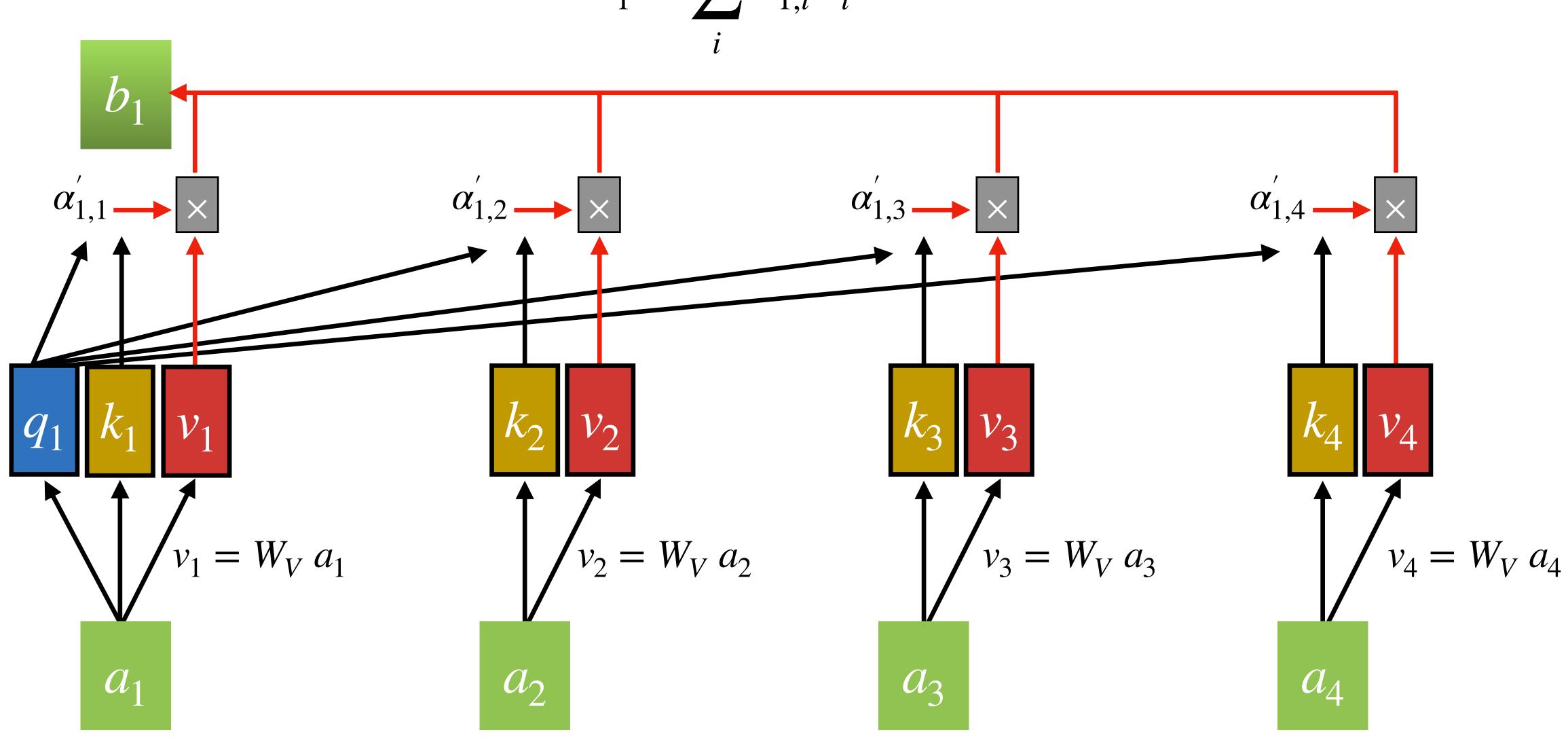
Denote how relevant each token are to a_1 !



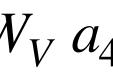
Use attention scores to extract information



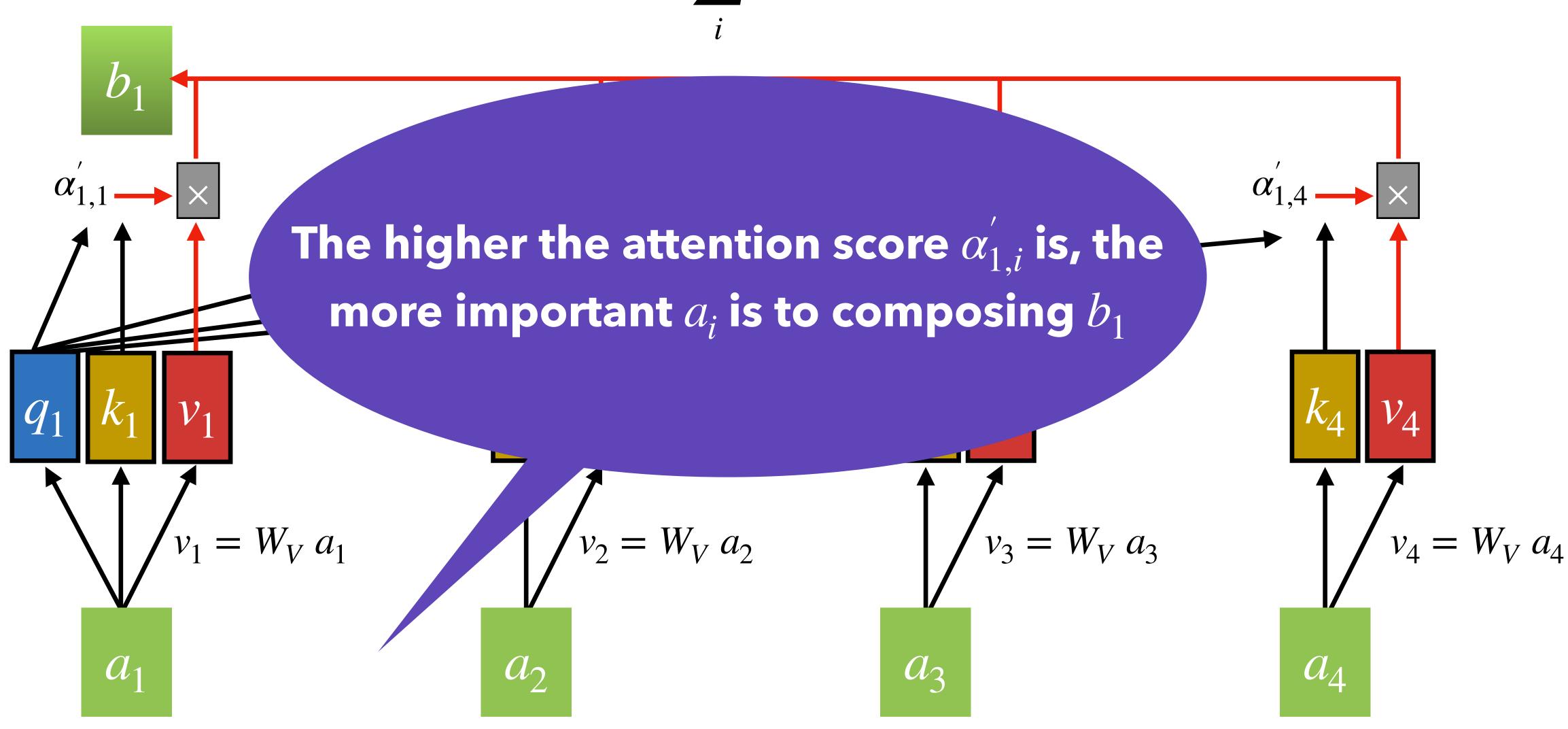
Use attention scores to extract information



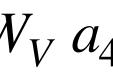
 $b_1 = \sum \alpha'_{1,i} v_i$



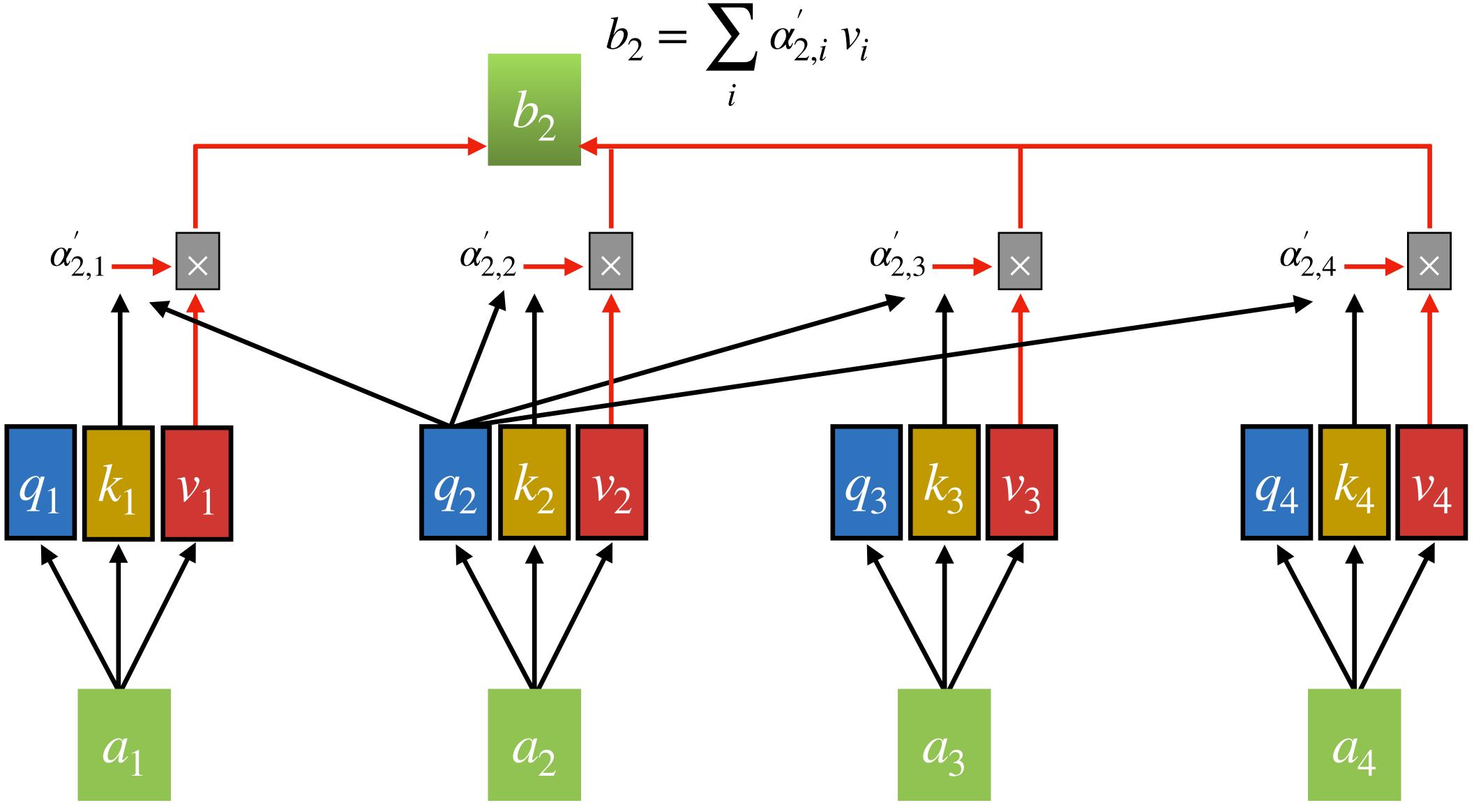
Use attention scores to extract information



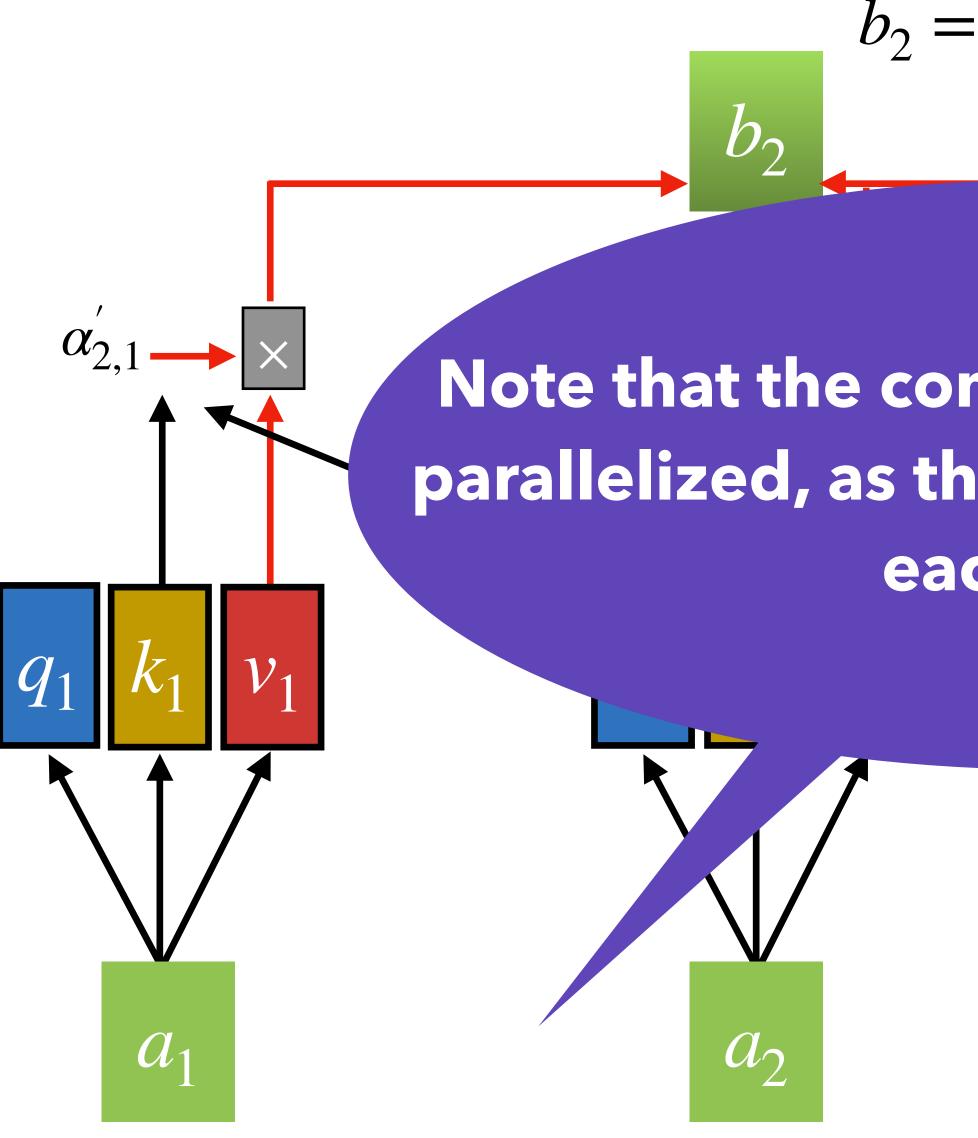
$$b_1 = \sum_{i} \alpha'_{1,i} v_i$$



Repeat the same calculation for all a_i to obtain b_i



Repeat the same calculation for all a_i to obtain b_i



 $b_2 = \sum_{i} \alpha'_{2,i} v_i$

Note that the computation of b_i can be parallelized, as they are independent to each other



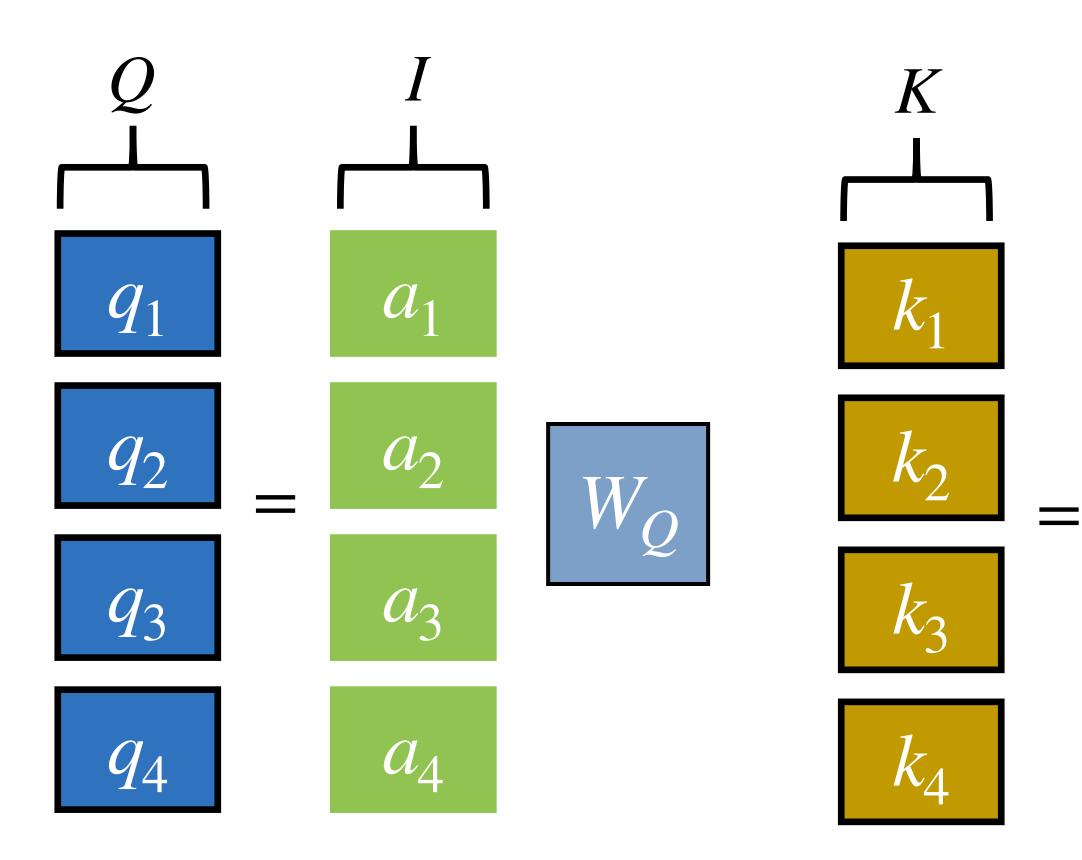
 $\alpha_{2, \ell}$

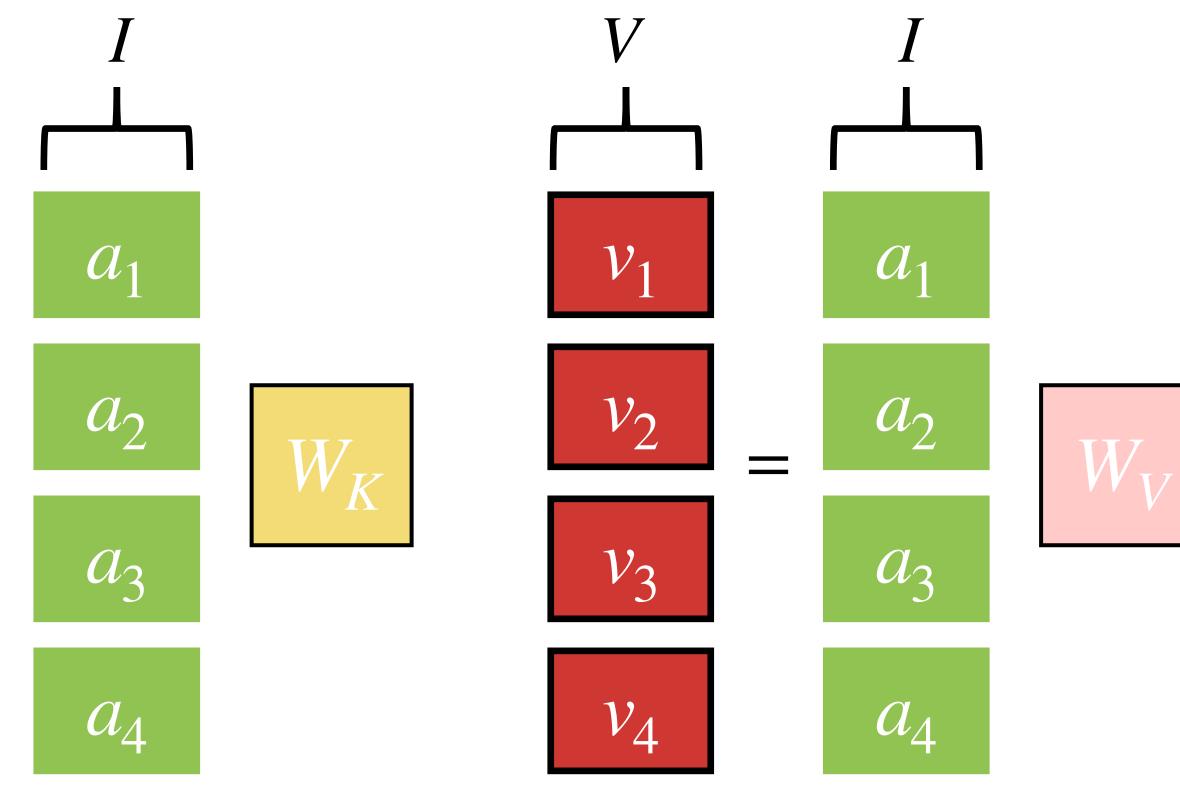
 q_4

 \mathcal{V}

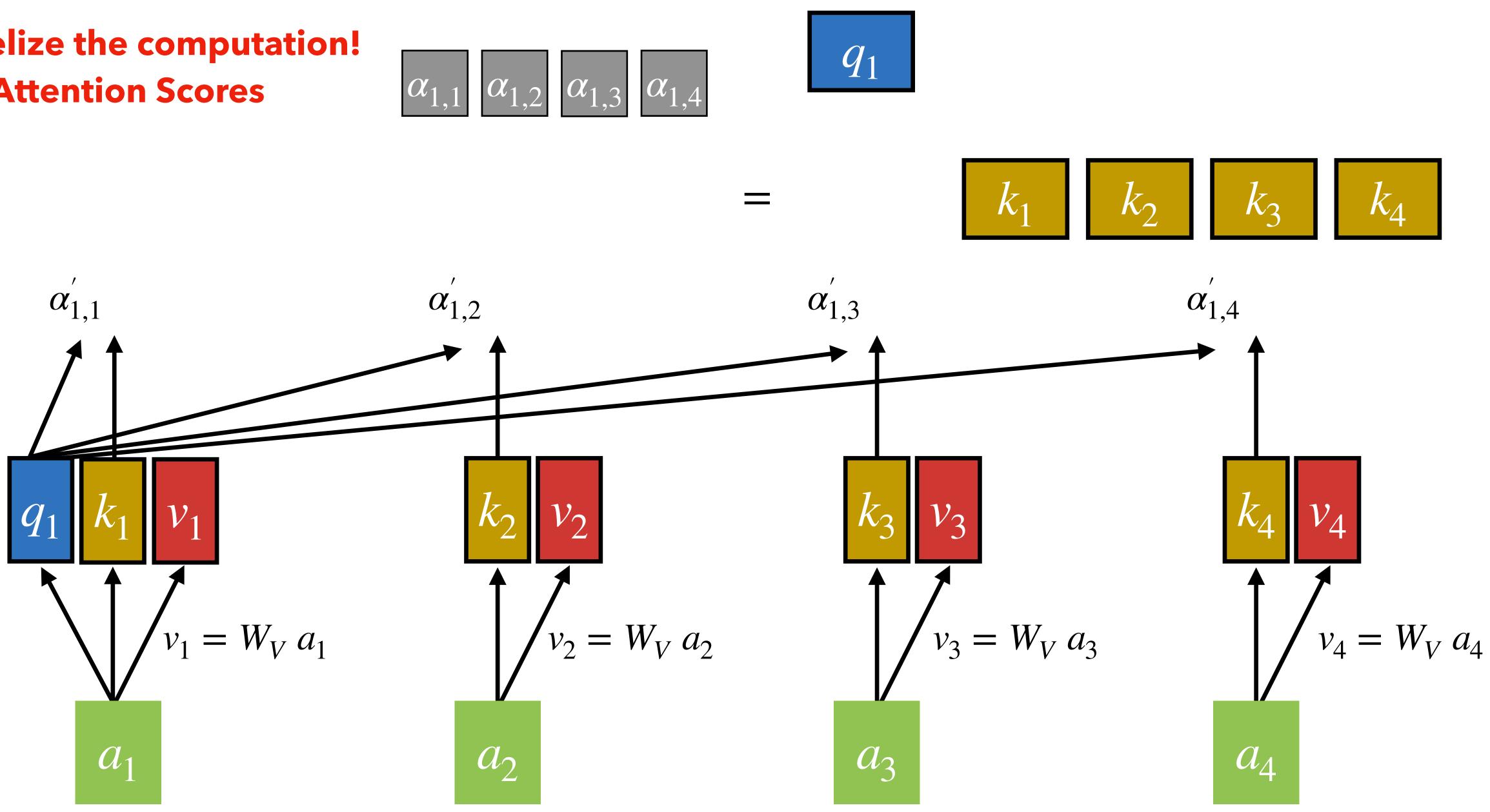
 a_4

Parallelize the computation! QKV

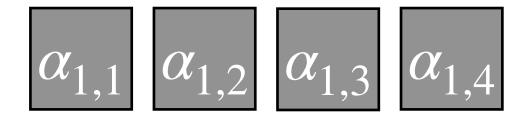




Parallelize the computation! $\alpha_{1,1}$ **Attention Scores** $\alpha_{1,2}$

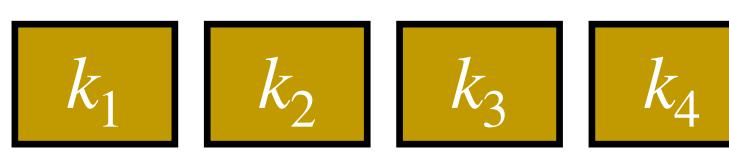


Parallelize the computation! Attention Scores



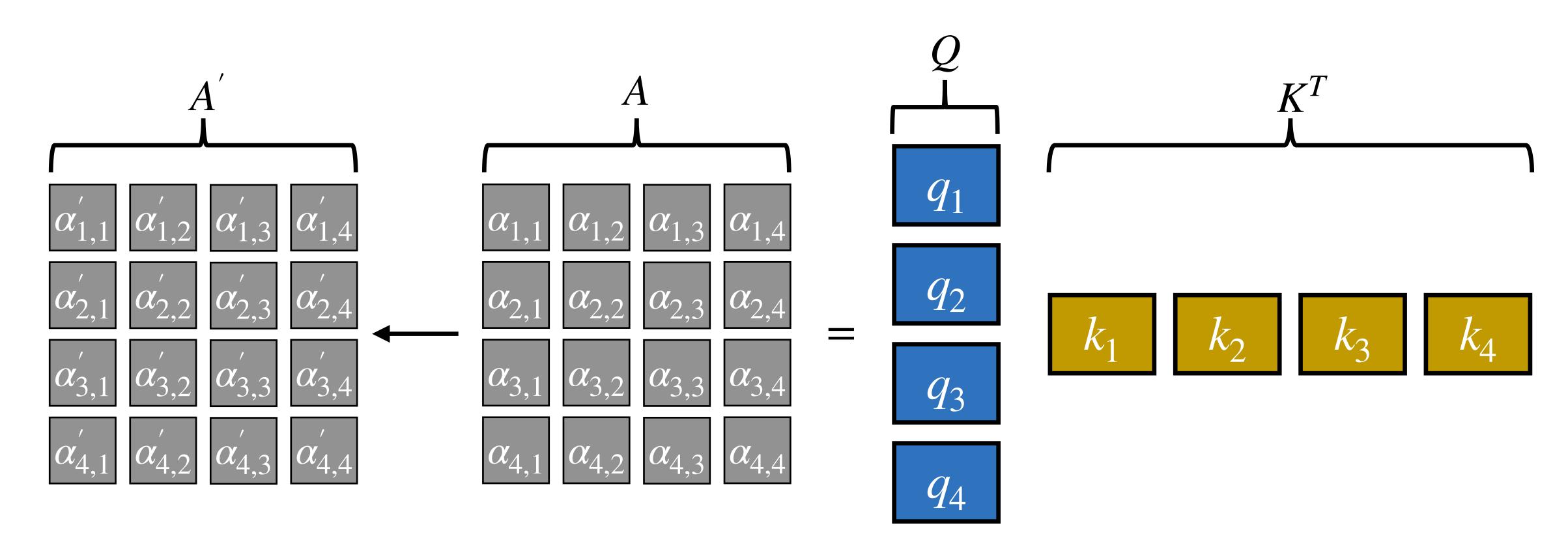


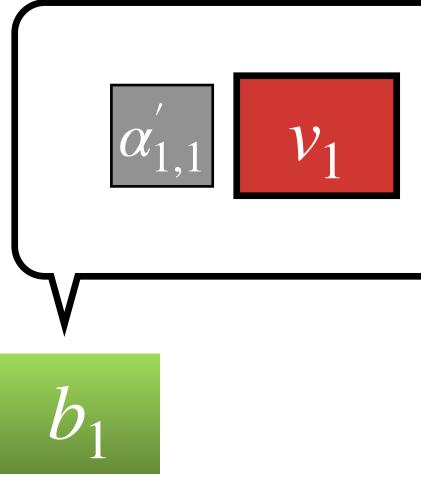


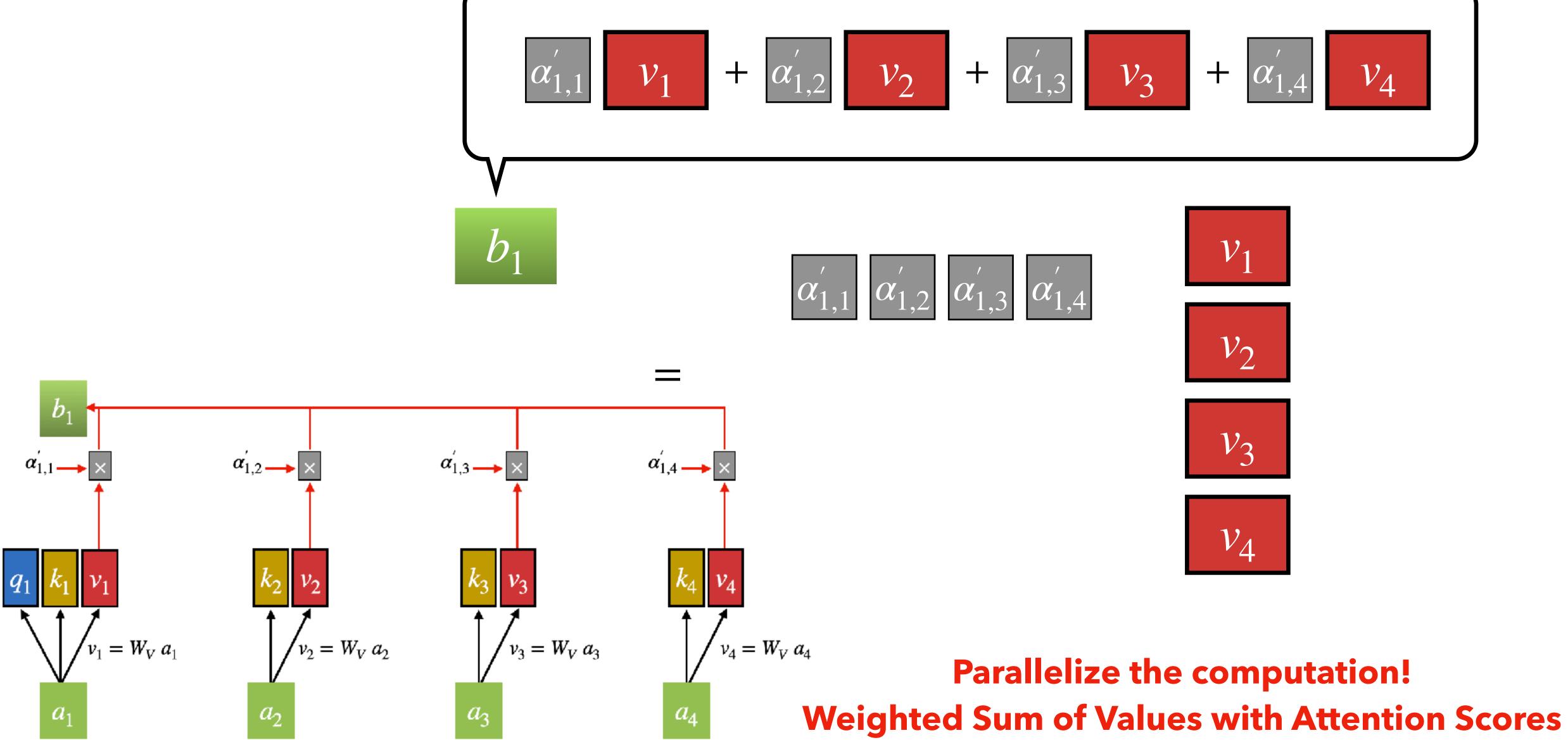




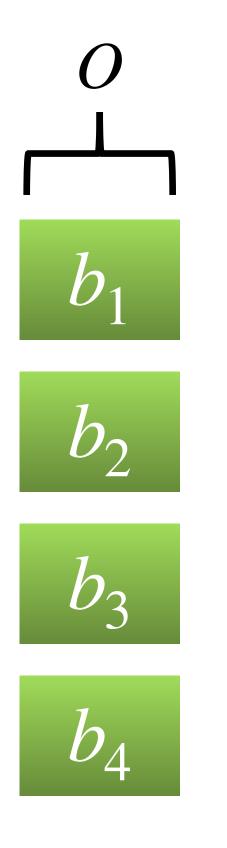
Parallelize the computation! Attention Scores

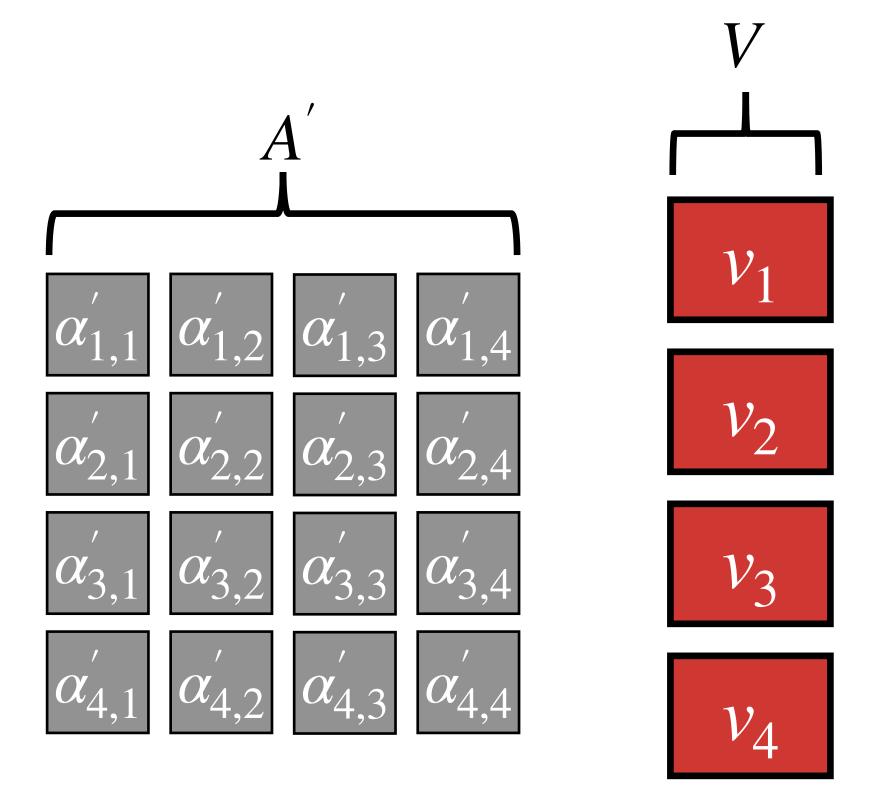






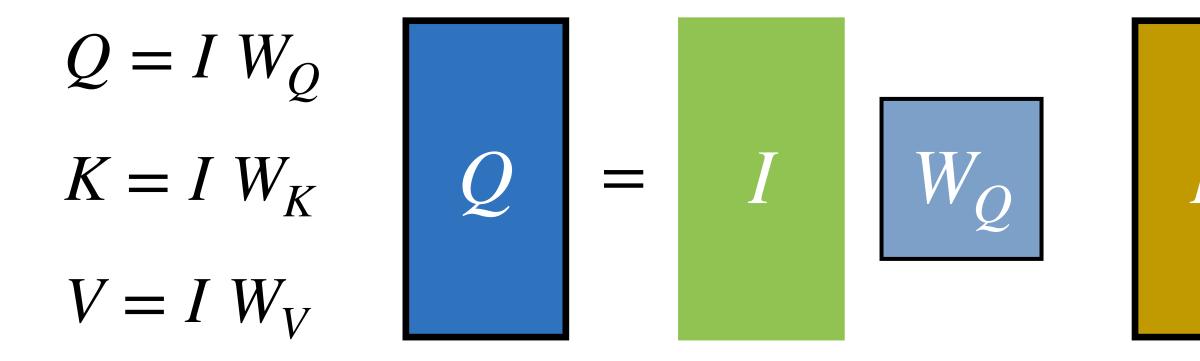
Parallelize the computation!



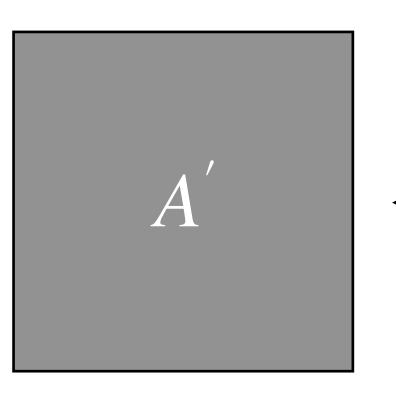


Parallelize the computation! Weighted Sum of Values with Attention Scores

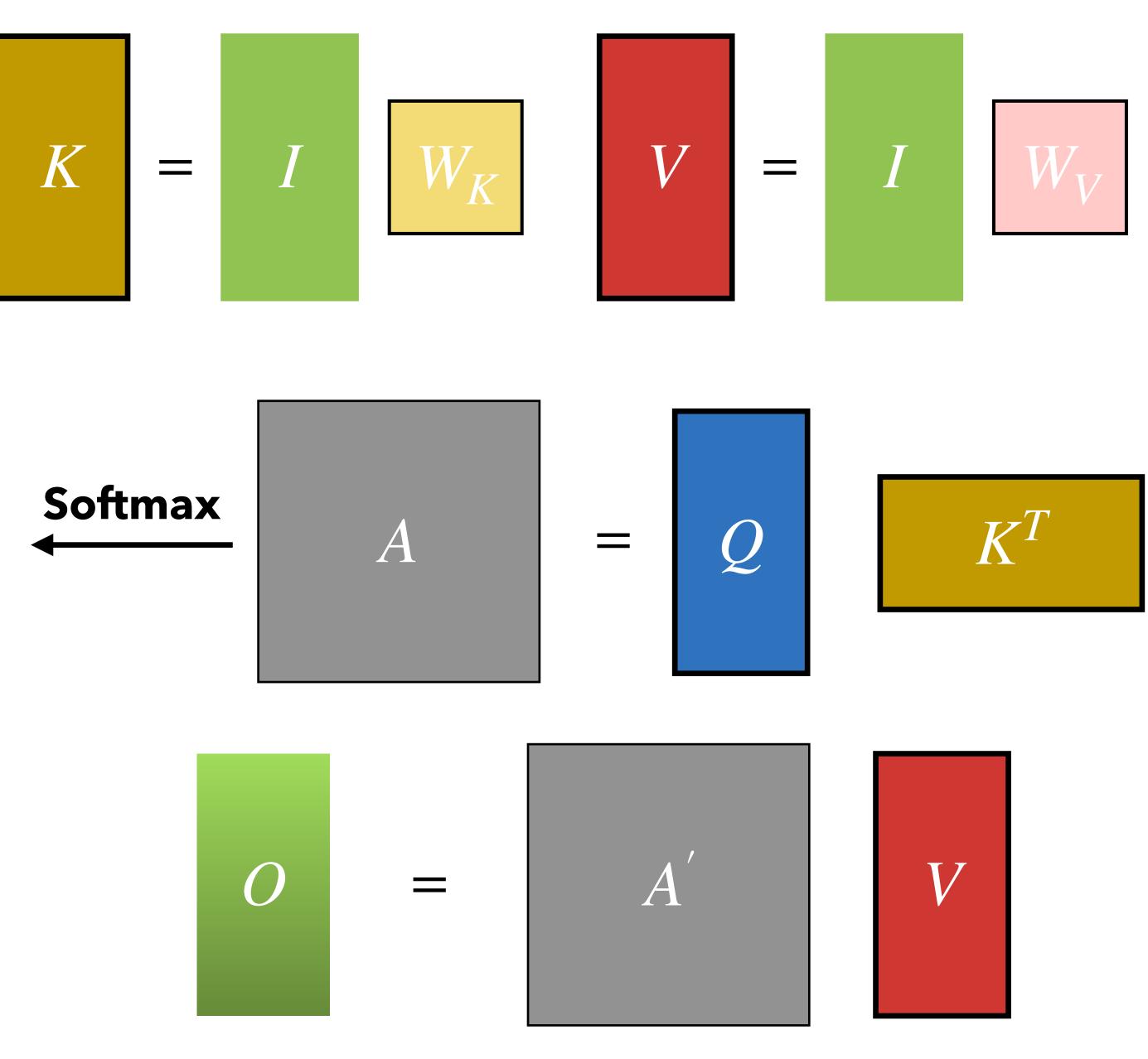




 $A = Q K^T$ $A = I W_Q (I W_K)^T = I W_Q W_K^T I^T$ $A' = \operatorname{softmax}(A)$



O = A' V



The Matrices Form of Self-Attention

- $Q = I W_Q$
- $K = I W_K$
- $V = I W_V$

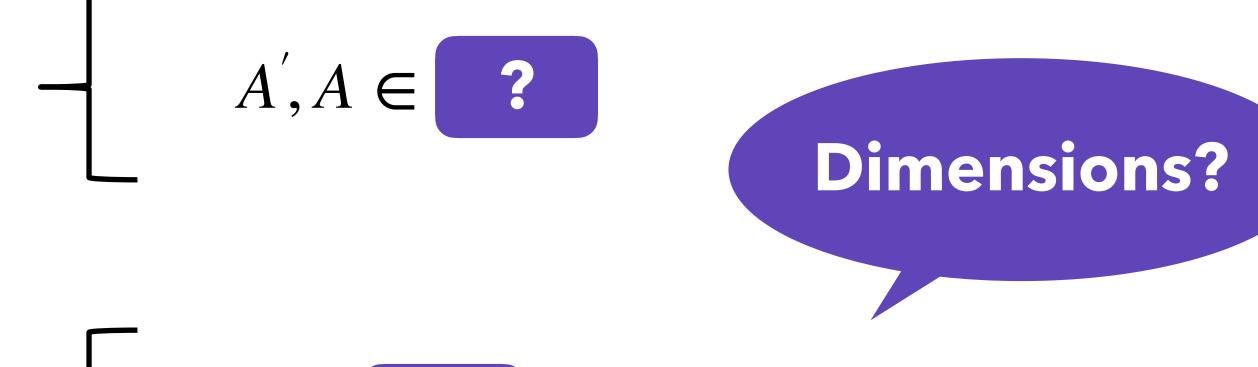
$$A = Q K^{T}$$

$$A = I W_{Q} (I W_{K})^{T} = I W_{Q} W_{K}^{T} I^{T}$$

$$A' = \text{softmax}(A)$$

$$O = A' V$$

 $I = \{a_1, \dots, a_n\} \in \mathbb{R}^{n \times d}, \text{ where } a_i \in \mathbb{R}^d$ $W_Q, W_K, W_V \in \mathbb{R}^{d \times d}$ $Q, K, V \in \ree$





The Matrices Form of Self-Attention

- $Q = I W_Q$
- $K = I W_K$
- $V = I W_V$

$$A = Q K^{T}$$

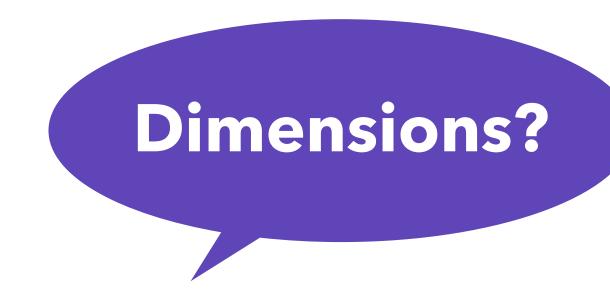
$$A = I W_{Q} (I W_{K})^{T} = I W_{Q} W_{K}^{T} I^{T}$$

$$A' = \text{softmax}(A)$$

$$O = A' V$$

 $\begin{bmatrix} I = \{a_1, \dots, a_n\} \in \mathbb{R}^{n \times d}, \text{ where } a_i \in \mathbb{R}^d \\ W_Q, W_K, W_V \in \mathbb{R}^{d \times d} \\ Q, K, V \in \mathbb{R}^{n \times d} \end{bmatrix}$

 $\neg A', A \in \mathbb{R}^{n \times n}$



 $O \in \mathbb{R}^{n \times d}$

Self-Attention: Summary

For each w_i , let $a_i = Ew_i$, where $E \in \mathbb{R}^{d \times |V|}$ is an embedding matrix.

$$\alpha_{i,j} = k_j q_i \qquad \qquad \alpha_{i,j} = -$$

3. Compute output for each word as weighted sum of values

 $b_i = \sum_{i=1}^{n}$

- Let $w_{1:n}$ be a sequence of words in vocabulary V, like Steve Jobs founded Apple.
- 1. Transform each word embedding with weight matrices W_O, W_K, W_V , each in $\mathbb{R}^{d \times d}$
 - $q_i = W_Q a_i$ (queries) $k_i = W_K a_i$ (keys) $v_i = W_V a_i$ (values)
- 2. Compute pairwise similarities between keys and queries; normalize with softmax $e^{\alpha_{i,j}}$

$$\sum_{j} e^{\alpha_{i,j}}$$

$$\sum \alpha_{i,j}^{'} v_{j}$$

Limitations and Solutions of Self-Attention



No Sequence Order

No Nonlinearities

Looking into the Future



Position Embedding

Adding Feed-forward Networks

Masking

Limitations and Solutions of Self-Attention



No Sequence Order

No Nonlinearities

Looking into the Future



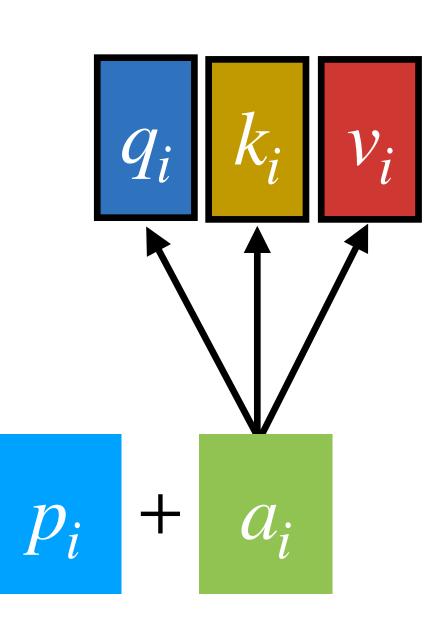
Position Embedding

Adding Feed-forward Networks

Masking

No Sequence Order \rightarrow Position Embedding

- All tokens in an input sequence are **simultaneously** fed into self-attention blocks. Thus, there's no difference between tokens at different positions.
 - We lose the position info!
- How do we bring the position info back, just like in RNNs? • **Representing each sequence index as a vector:** $p_i \in \mathbb{R}^d$, for $i \in \{1, ..., n\}$
- How to incorporate the position info into the self-attention blocks?
 - Just add the p_i to the input: $\hat{a}_i = a_i + p_i$
 - where a_i is the embedding of the word at index *i*.
 - In deep self-attention networks, we do this at the **first layer**.
 - We can also concatenate a_i and p_i , but more commonly we add them.



Position Representation Vectors via Sinusoids Sinusoidal Position Representations (from the original Transformer paper): concatenate sinusoidal functions of varying periods.

 $\sin(i/10000^{2*1/d})$ $\cos(i/10000^{2*1/d})$ Dimension $p_i =$ $sin(i/10000^{2*\frac{d}{2}/d})$ $cos(i/10000^{2*\frac{d}{2}/d})$ Index in the sequence https://timodenk.com/blog/linear-relationships-in-the-transformers-positional-encoding/

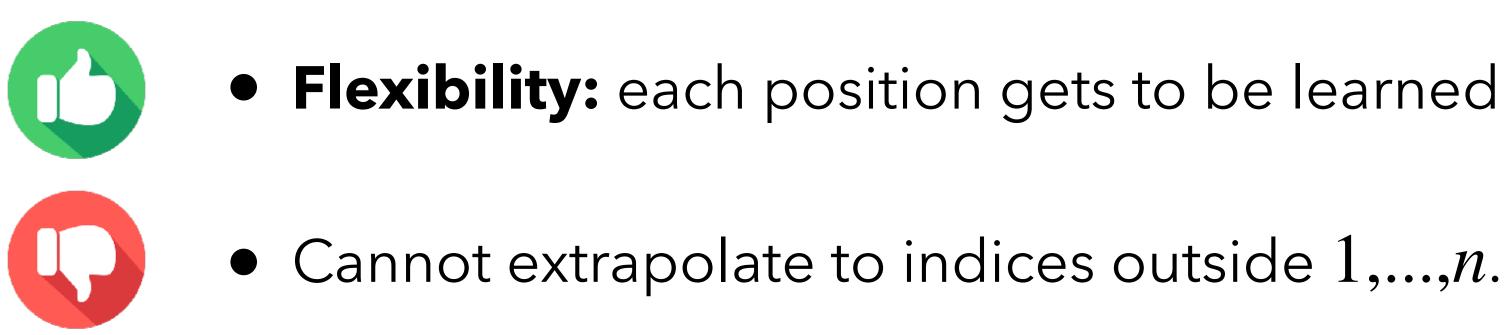
- Not learnable; also the extrapolation doesn't really work!



• Periodicity indicates that maybe "absolute position" isn't as important • Maybe can extrapolate to longer sequences as periods restart!

Learnable Position Representation Vectors

- Learn a matrix $p \in \mathbb{R}^{d \times n}$, and let each p_i be a column of that matrix
- Most systems use this method.



Sometimes people try more flexible representations of position:

- Relative linear position attention [Shaw et al., 2018]
- Dependency syntax-based position [Wang et al., 2019]

Learned absolute position representations: p_i contains learnable parameters.

• Flexibility: each position gets to be learned to fit the data

Limitations and Solutions of Self-Attention



No Sequence Order

No Nonlinearities

Looking into the Future



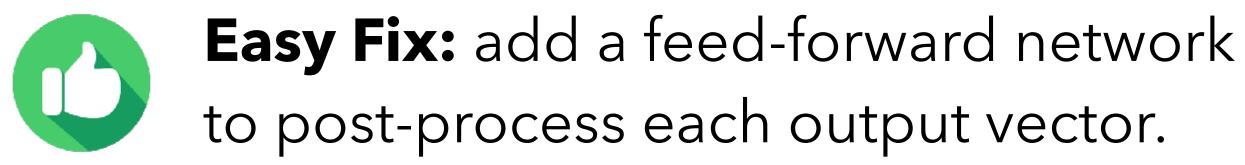
Position Embedding

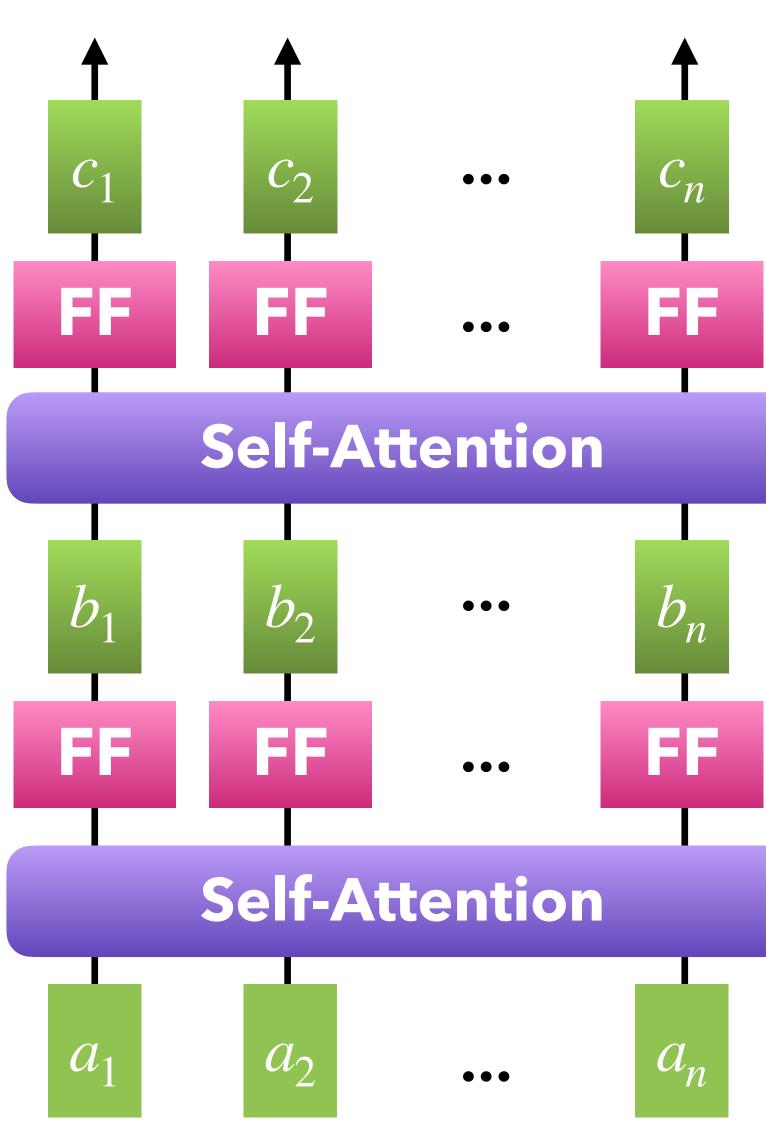
Adding Feed-forward Networks

Masking

No Nonlinearities \rightarrow Add Feed-forward Networks

There are **no element-wise nonlinearities** in self-attention; stacking more self-attention layers just re-averages value vectors.







Limitations and Solutions of Self-Attention



No Sequence Order

No Nonlinearities

Looking into the Future



Position Embedding

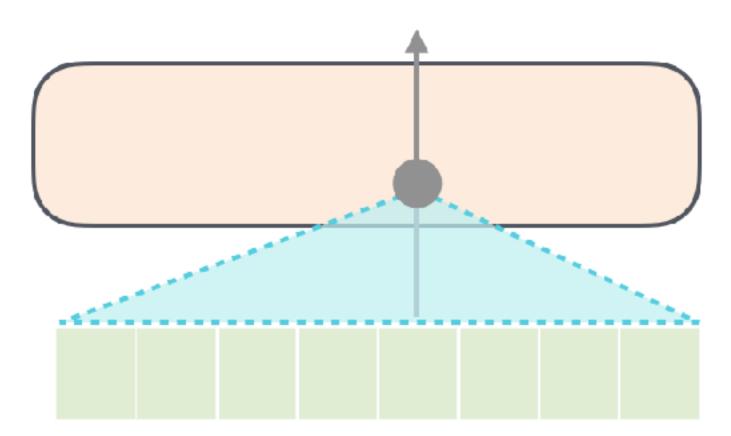
Adding Feed-forward Networks

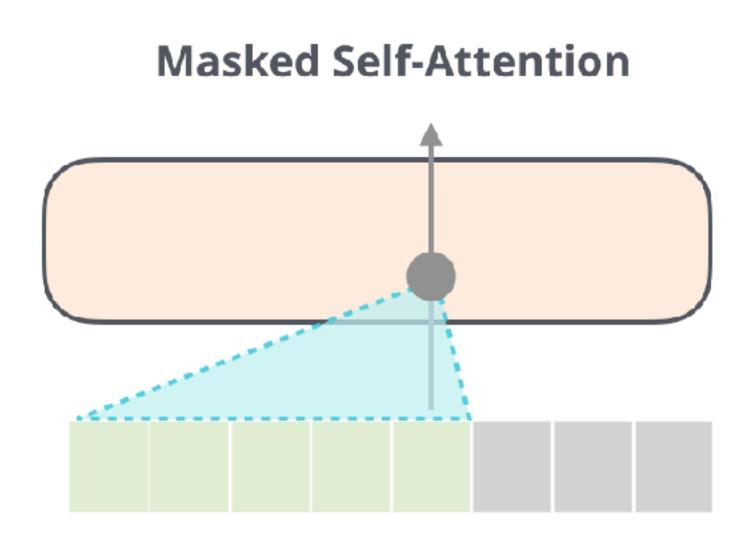
Masking

Looking into the Future \rightarrow Masking

 In decoders (language modeling, producing the next word given previous context), we need to ensure we **don't** peek at the future.

Self-Attention

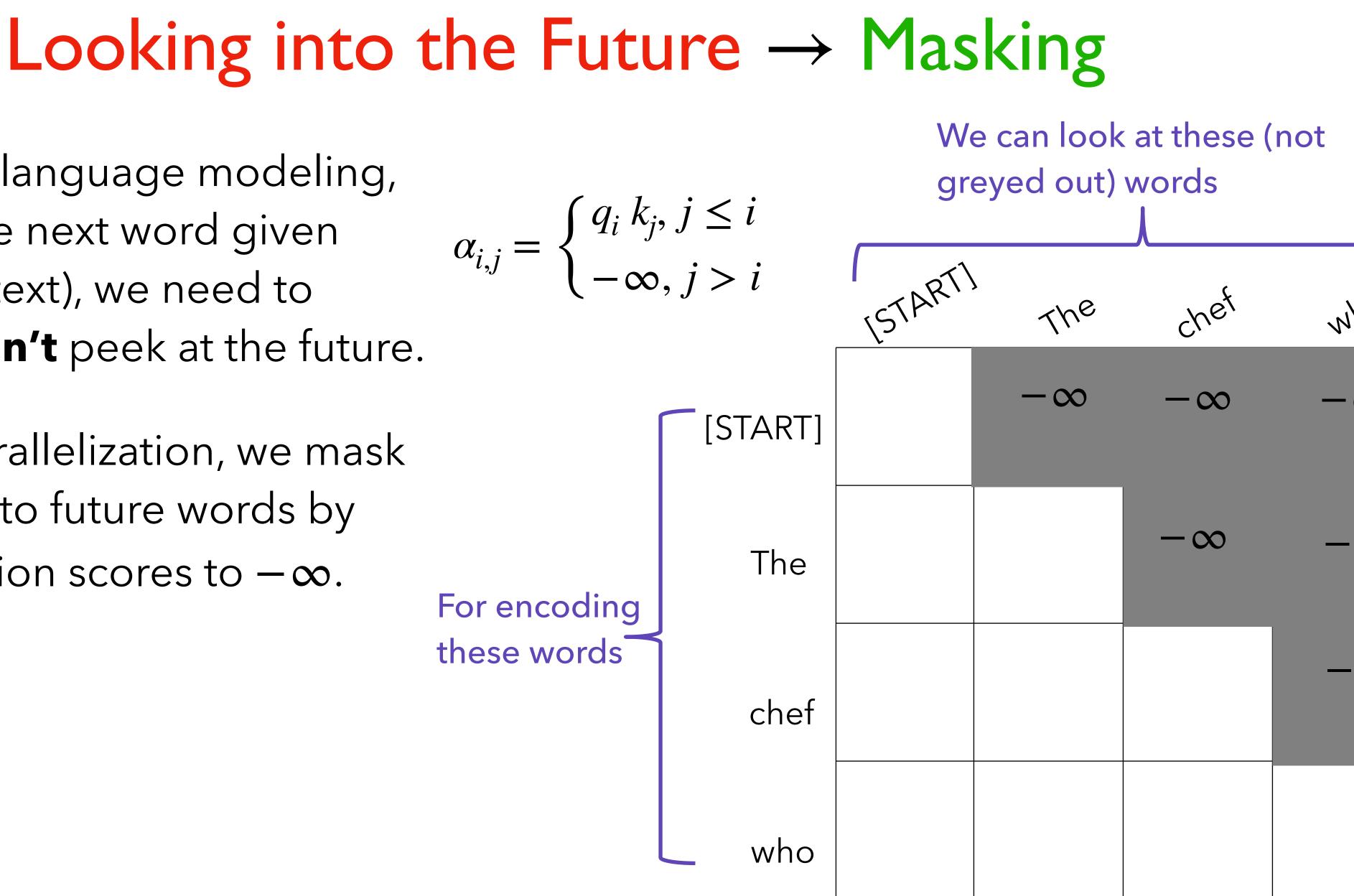


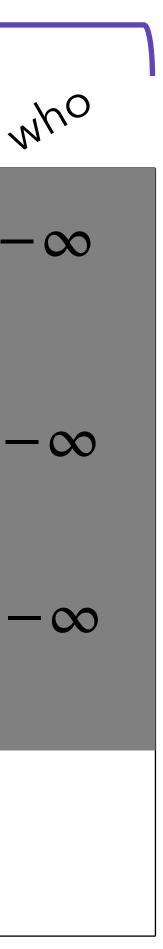


https://jalammar.github.io/illustrated-gpt2/



- In decoders (language modeling, producing the next word given previous context), we need to ensure we **don't** peek at the future.
- To enable parallelization, we mask out attention to future words by setting attention scores to $-\infty$.

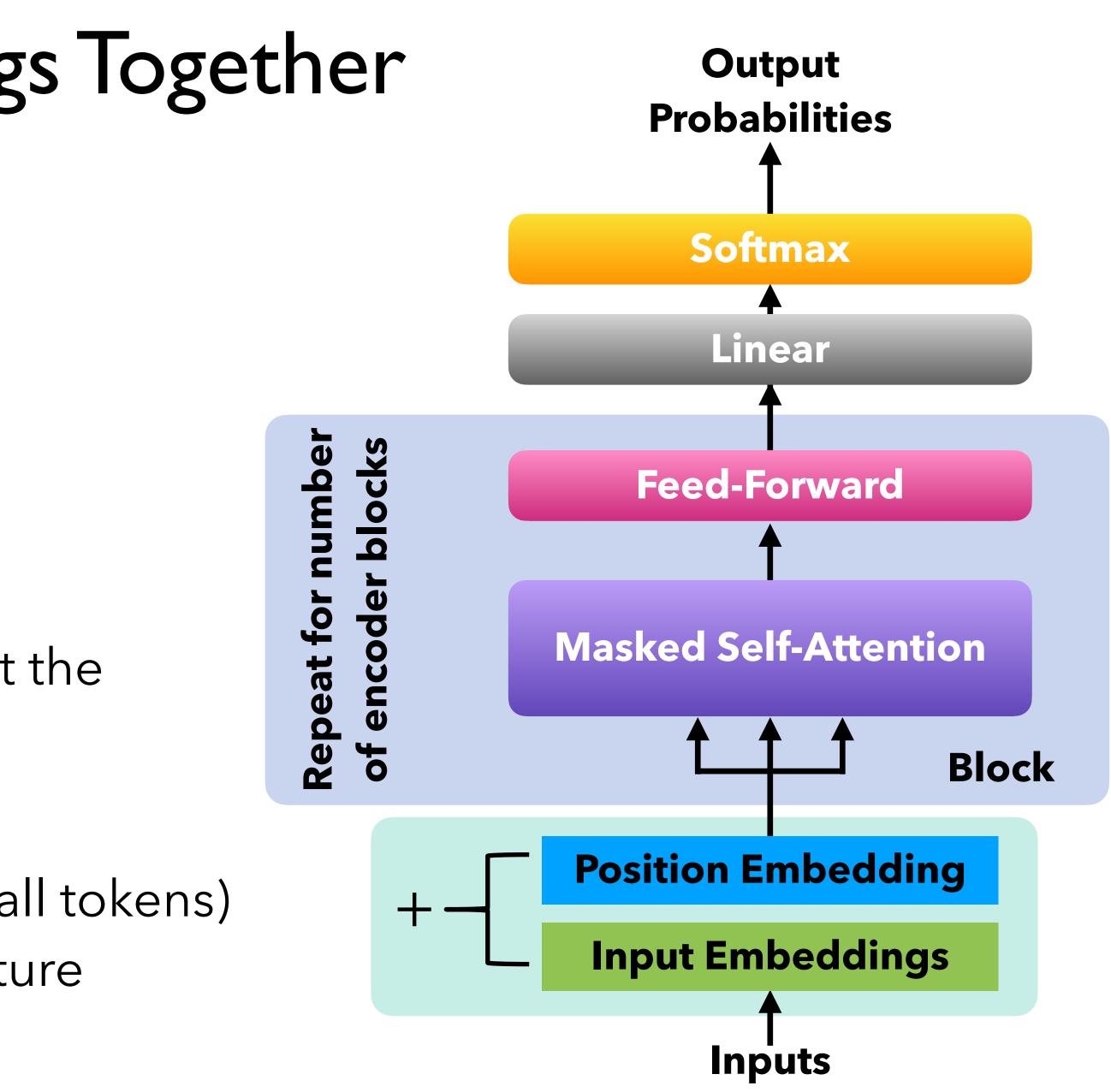




Now We Put Things Together

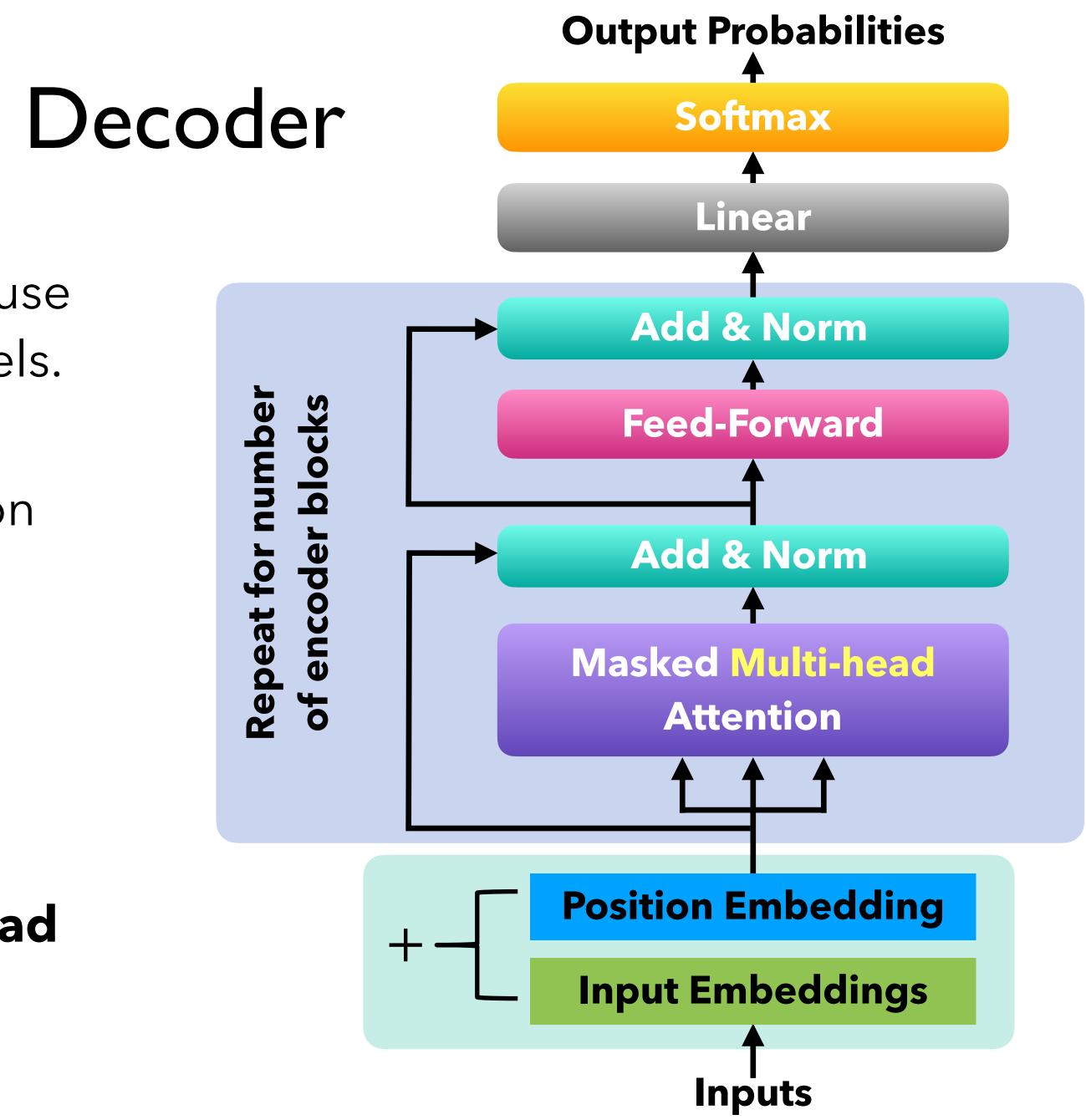
• Self-attention

- The basic computation
- Positional Encoding
 - Specify the sequence order
- Nonlinearities
 - Adding a feed-forward network at the output of the self-attention block
- Masking
 - Parallelize operations (looking at all tokens) while not leaking info from the future



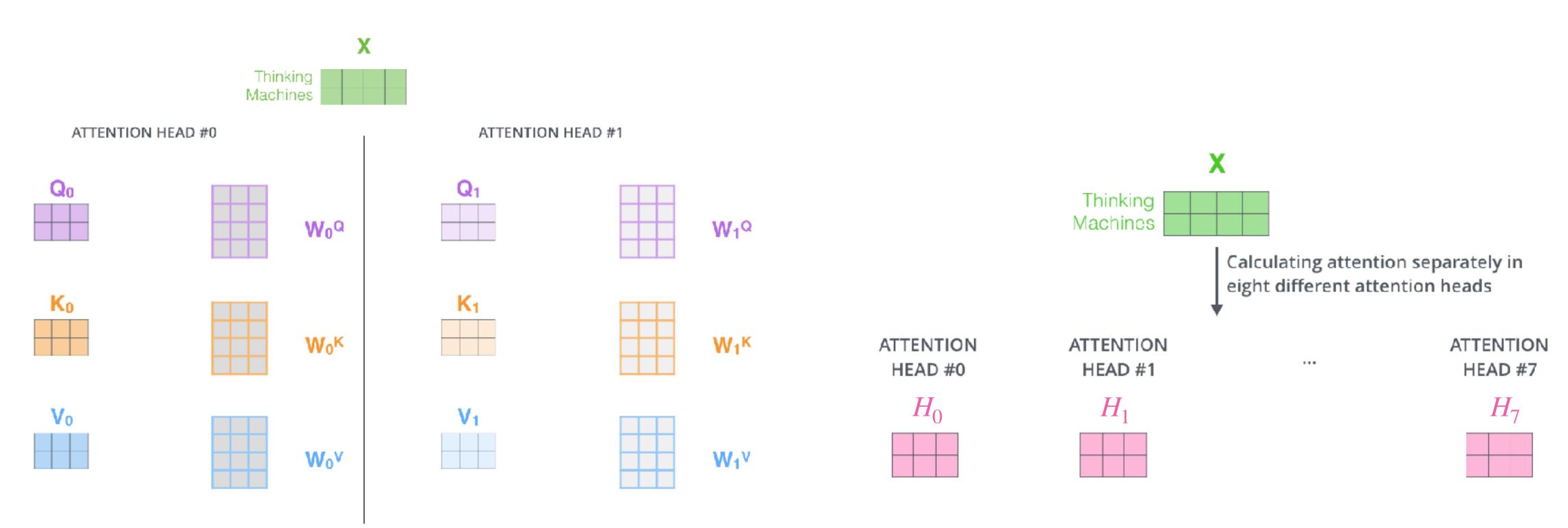
The Transformer Decoder

- A **Transformer decoder** is what we use to build systems like language models.
- It's a lot like our minimal self-attention architecture, but with a few more components.
 - Residual connection ("Add")
 - Layer normalization ("Norm")
- Replace self-attention with multi-head self-attention.



Multi-head Attention

- It is better to use multiple attention functions instead of one!
 - Each attention function ("head") can focus on different positions.

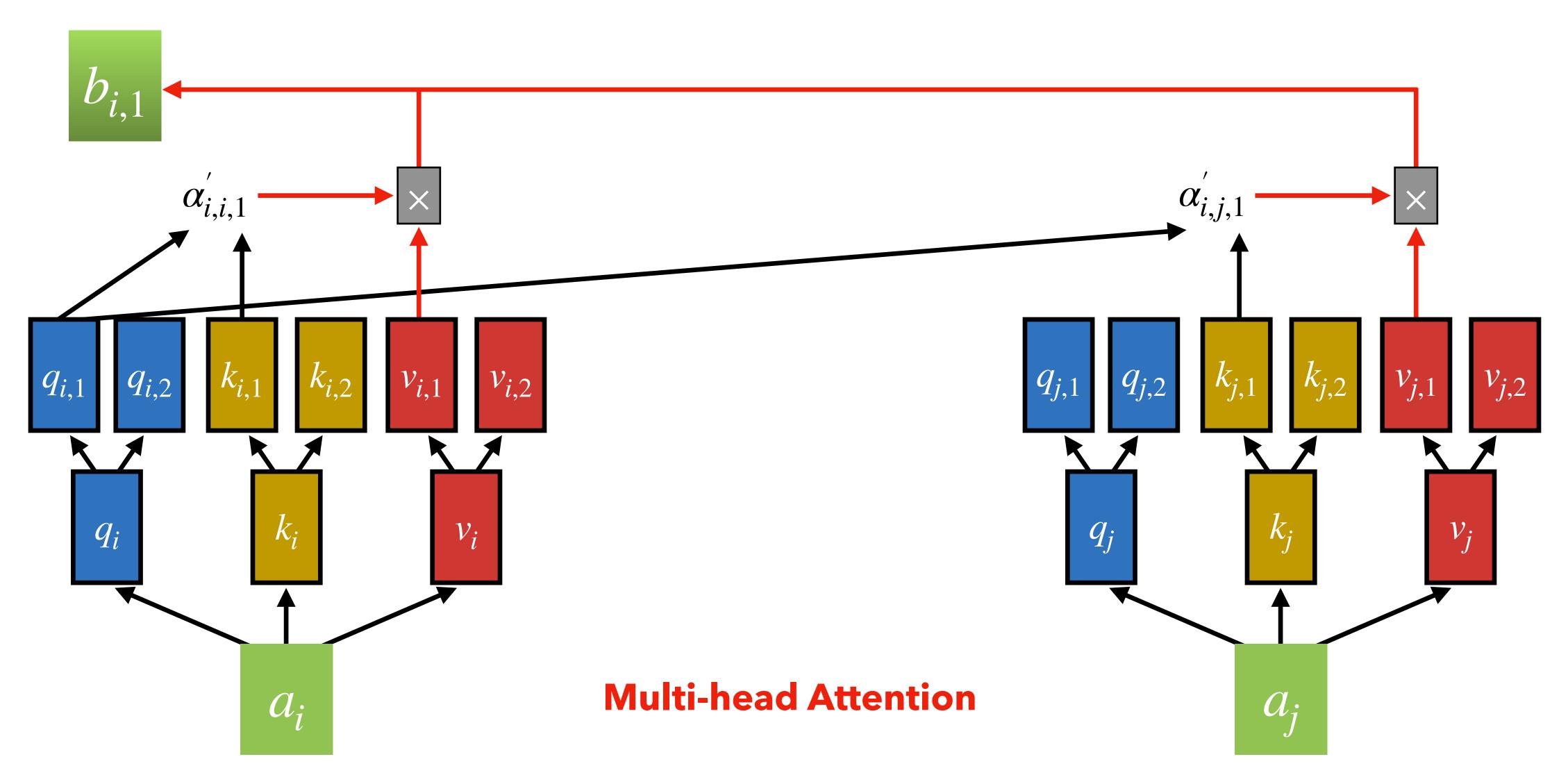


"The Beast with Many Heads"

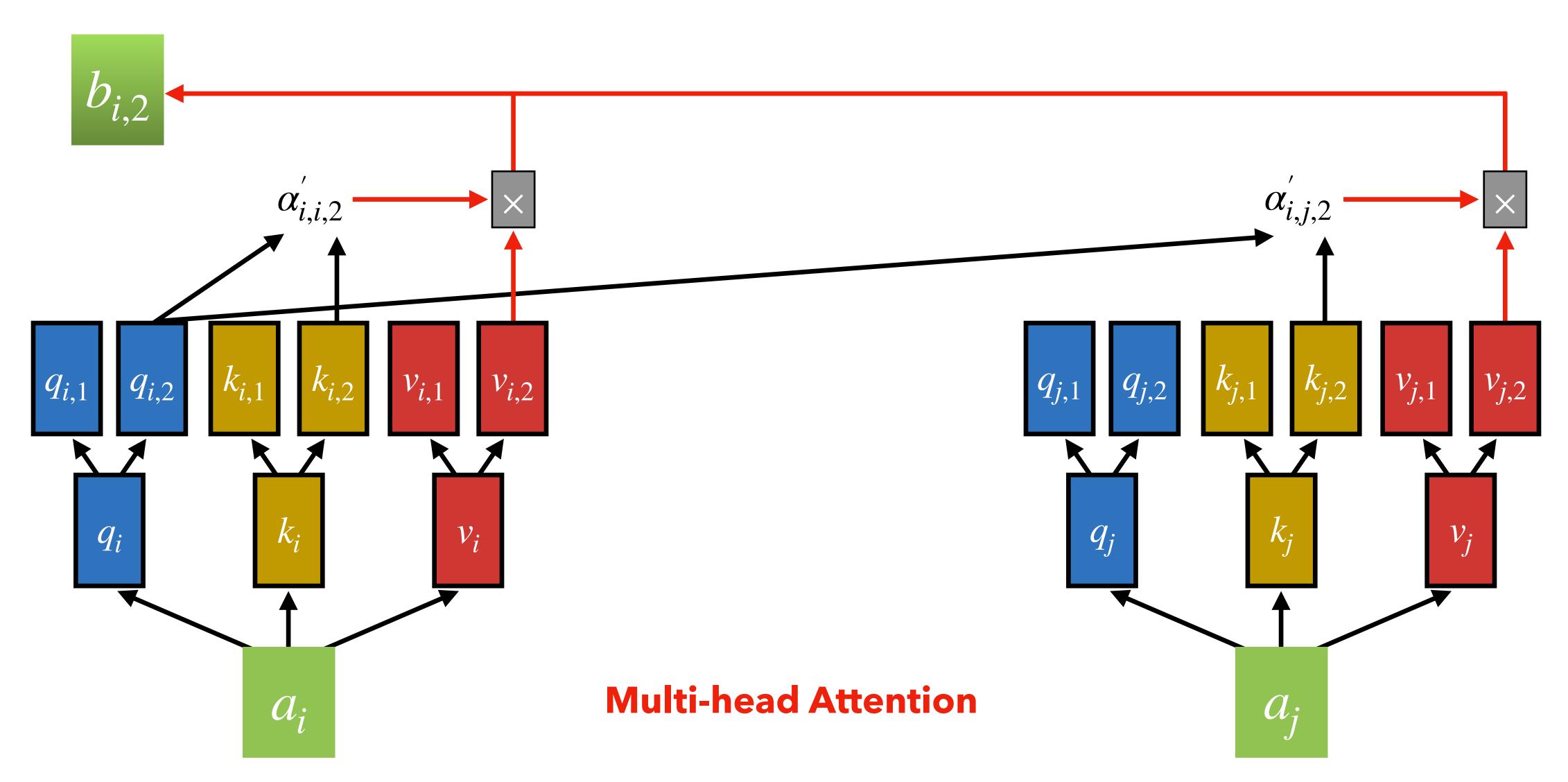
https://jalammar.github.io/illustrated-transformer/

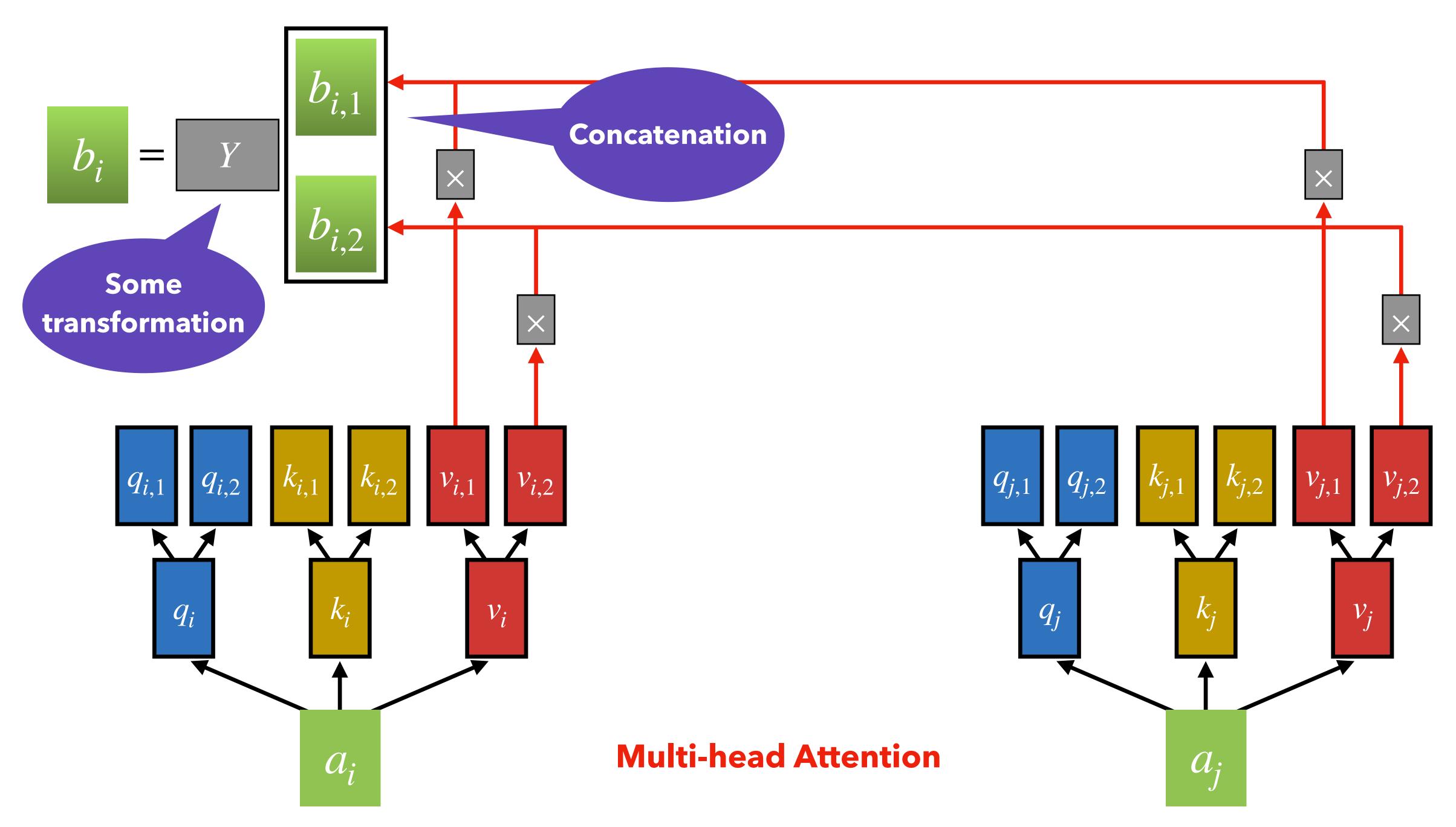


Multi-Head Attention: Walk-through



Multi-Head Attention: Walk-through





Recall the Matrices Form of Self-Attention

$$Q = I W_Q$$

- $K = I W_K$
- $V = I W_V$

$$A = Q K^{T}$$
$$A = I W_{Q} (I W_{K})^{T} = I W_{Q} W_{K}^{T} I^{T}$$
$$A' = \text{softmax}(A)$$

$$O = A' V$$

 $I = \{a_1, \dots, a_n\} \in \mathbb{R}^{n \times d}, \text{ where } a_i \in \mathbb{R}^d$ $W_Q, W_K, W_V \in \mathbb{R}^{d \times d}$ $Q, K, V \in \mathbb{R}^{n \times d}$

 $\neg \qquad A', A \in \mathbb{R}^{n \times n}$

 $\mathbf{O} \in \mathbb{R}^{n \times d}$

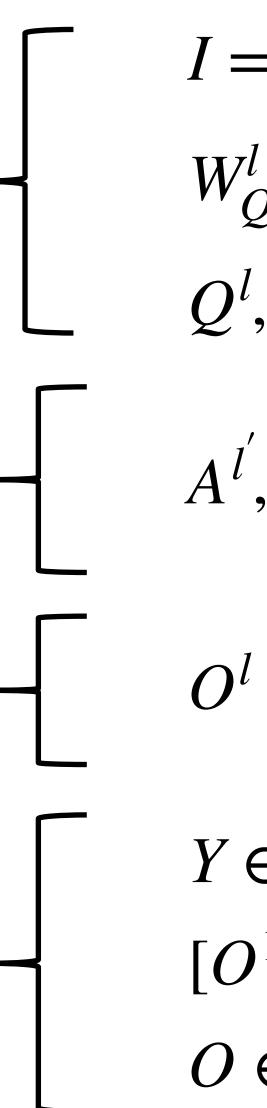
Multi-head Attention in Matrices

- Multiple attention "heads" can be defined via multiple W_Q, W_K, W_V matrices
 Let W^l_Q, W^l_K, W^l_V ∈ ℝ^{d×^d/h}, where h is the number of attention heads, and l ranges
- Let W_Q^l , W_K^l , $W_V^l \in \mathbb{R}^{d \times \frac{d}{h}}$, where h is the from 1 to h.
- Each attention head performs attention independently: • $O^{l} = \operatorname{softmax}(I W_{Q}^{l} W_{K}^{l} I^{T}) I W_{V}^{l}$
- Concatenating different O^l from different attention heads. • $O = [O^1; \ldots; O^n] Y$, where $Y \in \mathbb{R}^{d \times d}$

The Matrices Form of Multi-head Attention

 $Q^l = I W_O^l$ $K^l = I \ W^l_K$ $V^l = I W^l_V$ $A^l = Q^l K^{l^T}$ $A^{l'} = \operatorname{softmax}(A^l)$ $O^{l} = A^{l'} V^{l}$

 $O = [O^1; \ldots; O^h] Y$



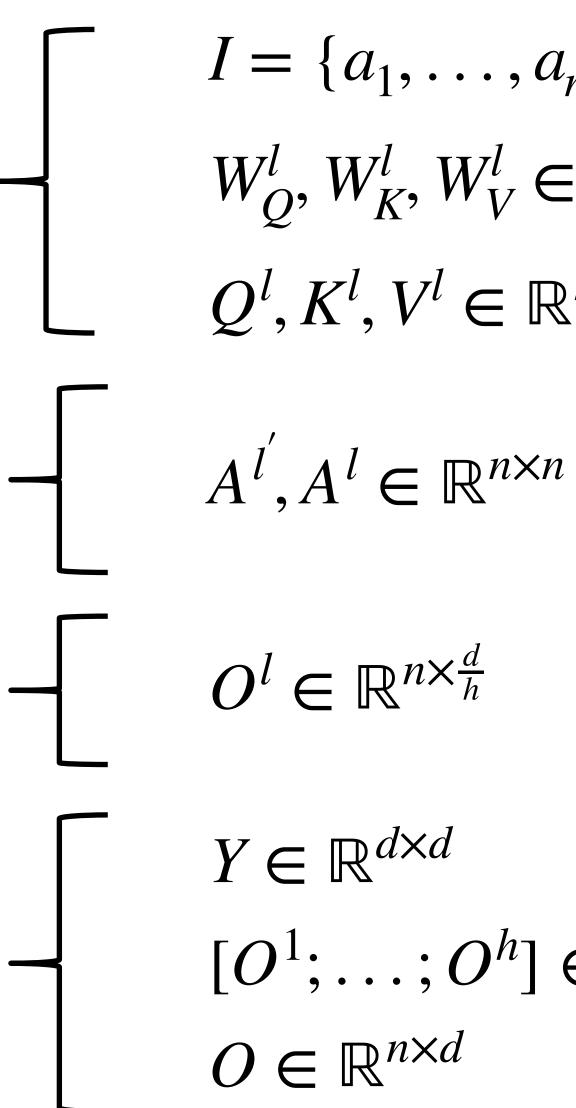
- $I = \{a_1, \dots, a_n\} \in \mathbb{R}^{n \times d}, \text{ where } a_i \in \mathbb{R}^d$ $W_Q^l, W_K^l, W_V^l \in \mathbb{R}^{d \times \frac{d}{h}}$ $Q^l, K^l, V^l \in \ree$ $- \qquad A^{l'}, A^l \in \mathbb{R} ?$ **Dimensions?** $O^l \in \mathbb{R}$?
 - $Y \in \mathbb{R}^{d \times d}$ $[O^1; \dots; O^h] \in ?$ $O \in \mathbb{R}$?



The Matrices Form of Multi-head Attention

 $Q^l = I W_O^l$ $K^l = I \ W^l_{\kappa}$ $V^l = I W^l_V$ $A^l = Q^l K^{l^T}$ $A^{l'} = \operatorname{softmax}(A^l)$ $O^l = A^{l'} V^l$

 $O = [O^1; \ldots; O^h] Y$



- $$\begin{split} I &= \{a_1, \dots, a_n\} \in \mathbb{R}^{n \times d}, \text{ where } a_i \in \mathbb{R}^d \\ W_Q^l, W_K^l, W_V^l \in \mathbb{R}^{d \times \frac{d}{h}} \\ Q^l, K^l, V^l \in \mathbb{R}^{n \times \frac{d}{h}} \end{split}$$
- $O^l \in \mathbb{R}^{n \times \frac{d}{h}}$

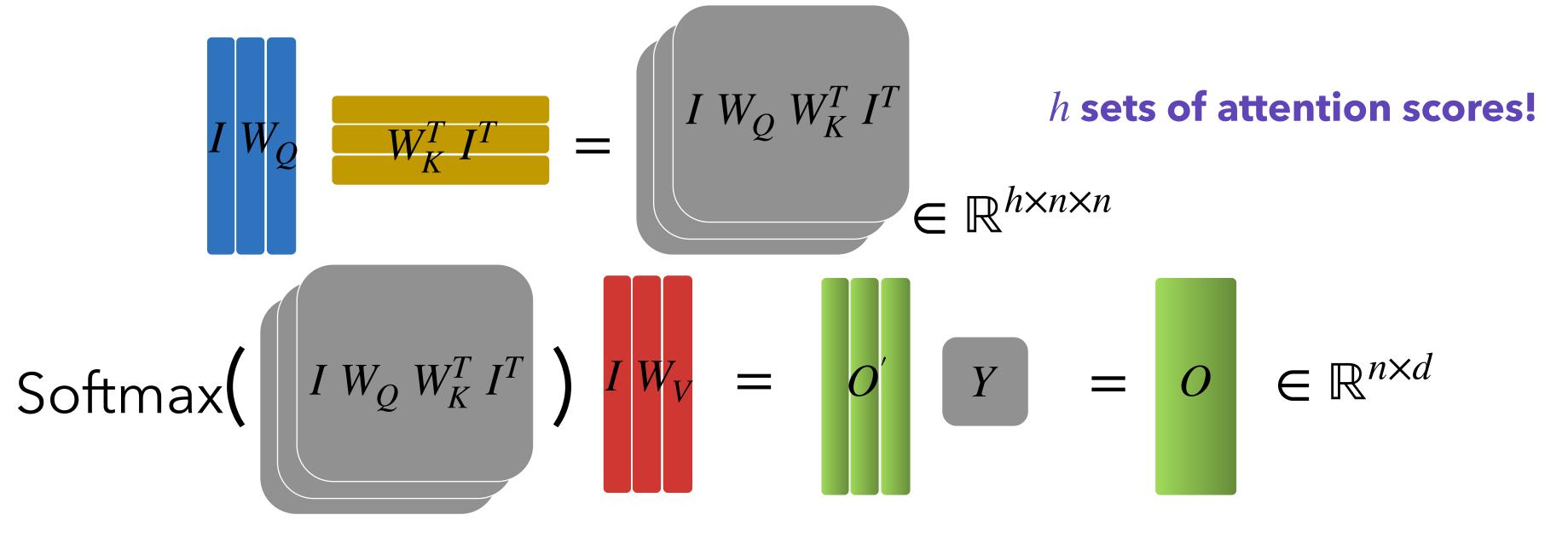


 $Y \in \mathbb{R}^{d \times d}$ $[O^1; \dots; O^h] \in \mathbb{R}^{n \times d}$ $O \in \mathbb{R}^{n \times d}$



Multi-head Attention is Computationally Efficient

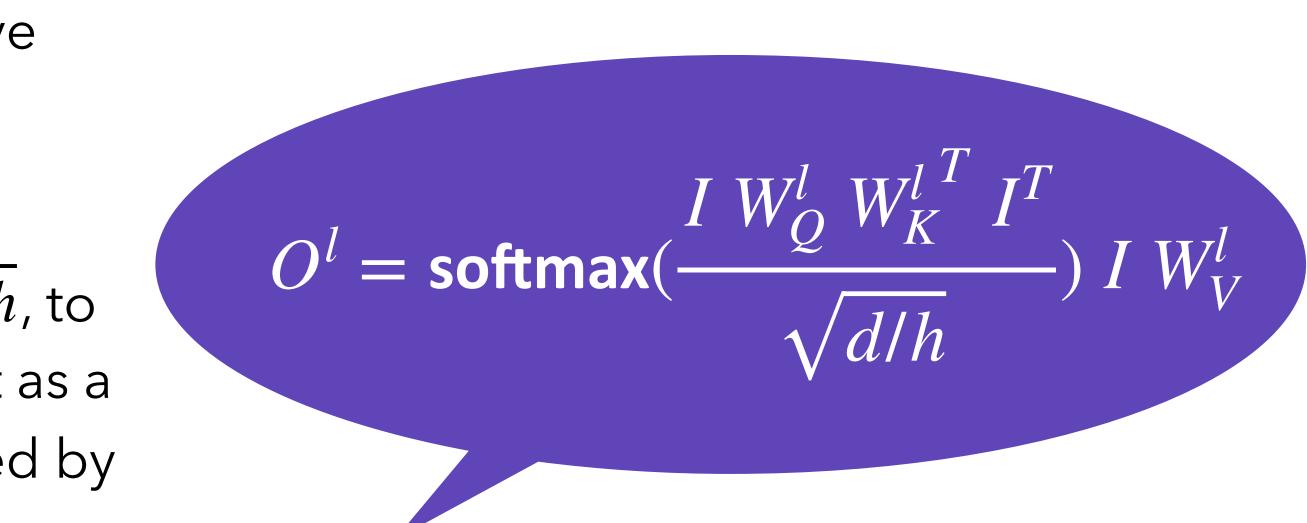
- Even though we compute h many attention heads, it's not more costly.
 - We compute $I W_O \in \mathbb{R}^{n \times d}$, and then reshape to $\mathbb{R}^{n \times h \times \frac{d}{h}}$.
 - Likewise for $I W_K$ and $I W_V$.
 - Then we transpose to $\mathbb{R}^{h \times n \times \frac{d}{h}}$; now the head axis is like a batch axis.



• Almost everything else is identical. All we need to do is to reshape the tensors!

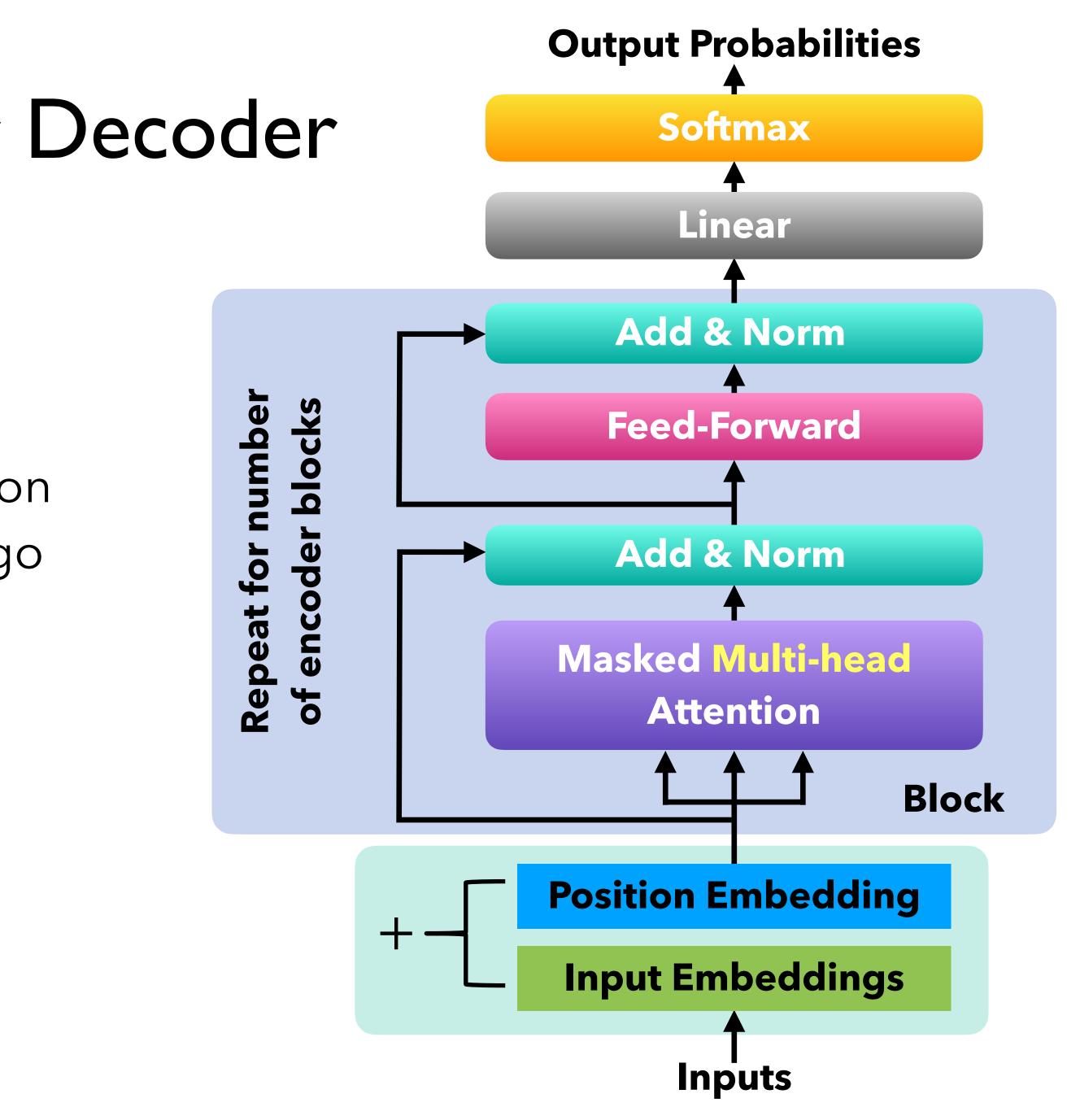
Scaled Dot Product

- "Scaled Dot Product" attention aids in training.
- When dimensionality *d* becomes large, dot products between vectors tend to become large.
 - Because of this, inputs to the softmax function can be large, making the gradients small.
- Instead of the self-attention function we've seen:
 - $O^l = \operatorname{softmax}(I W_Q^l W_K^{l^T} I^T) I W_V^l$
- We divide the attention scores by $\sqrt{d/h}$, to stop the scores from becoming large just as a function of d/h (the dimensionality divided by the number of heads).



The Transformer Decoder

- Now that we've replaced self-attention with multi-head self-attention, we'll go through two **optimization tricks**:
 - Residual connection ("Add")
 - Layer normalization ("Norm")



• Residual connections are a trick to help models train better.

• Instead of $X^{(i)} = \text{Layer}(X^{(i-1)})$ (where *i* represents the layer)

$$X^{(i-1)} - Layer$$
We let $X^{(i)} = X^{(i-1)} + Layer(X^{(i-1)})$
the previous layer)
$$X^{(i-1)} - Layer + Layer$$

- Gradient is great through the residual connection; it's 1!
- Bias towards the identity function!

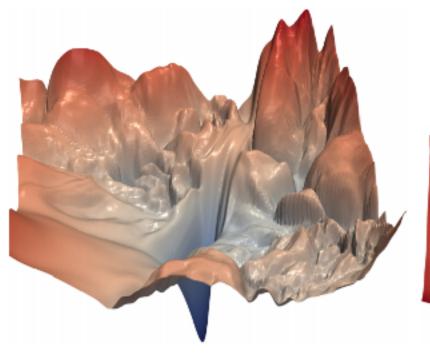
Residual Connections

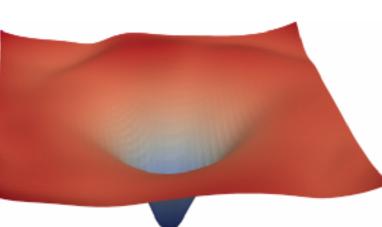
 $X^{(i)}$

(so we only have to learn "the residual" from

 $X^{(i)}$







[no residuals]

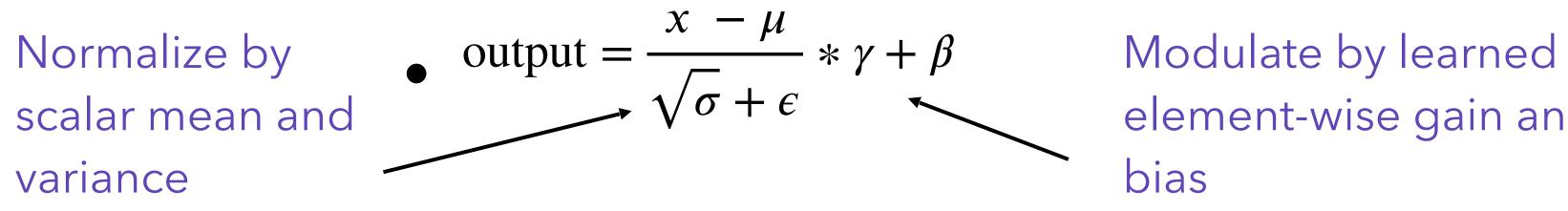
[residuals]

[Loss landscape visualization, Li et al., 2018, on a ResNet]

Layer Normalization

- Layer normalization is a trick to help models train faster.
- and standard deviation within each layer.
 - LayerNorm's success may be due to its normalizing gradients [Xu et al., 2019]
- Let $x \in \mathbb{R}^d$ be an individual (word) vector in the model. • Let $\mu = \sum_{j=1}^{n} x_{j}$; this is the mean; $\mu \in \mathbb{R}$. • Let $\sigma = \sqrt{\frac{1}{d} \sum_{j=1}^{d} (x_j - \mu)^2}$; this is the standard deviation; $\sigma \in \mathbb{R}$.
- Let $\gamma \in \mathbb{R}^d$ and $\beta \in \mathbb{R}^d$ be learned "gain" and "bias" parameters. (Can omit!)
- Then layer normalization computes:

scalar mean and variance

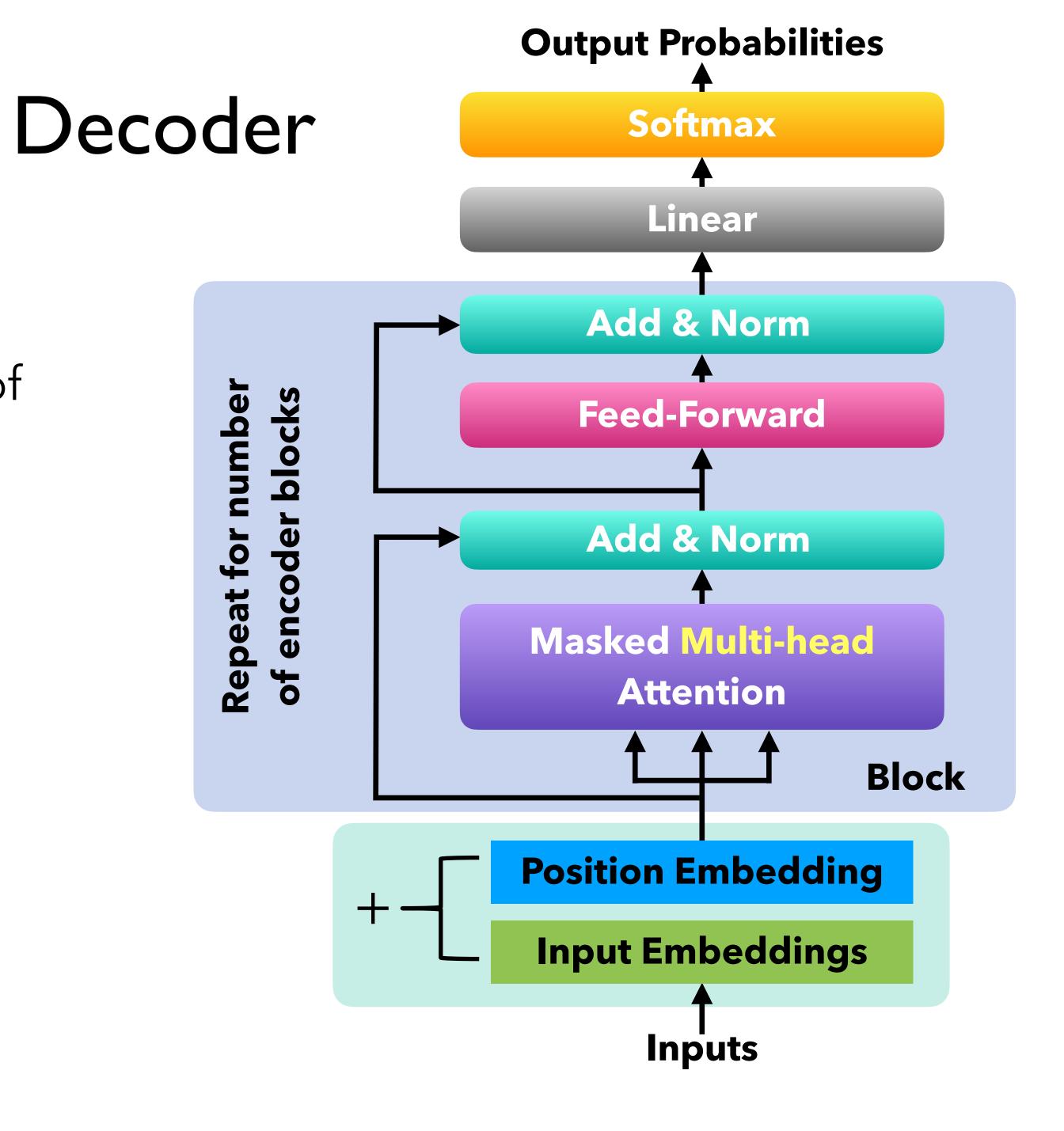


Idea: cut down on uninformative variation in hidden vector values by normalizing to unit mean

element-wise gain and bias

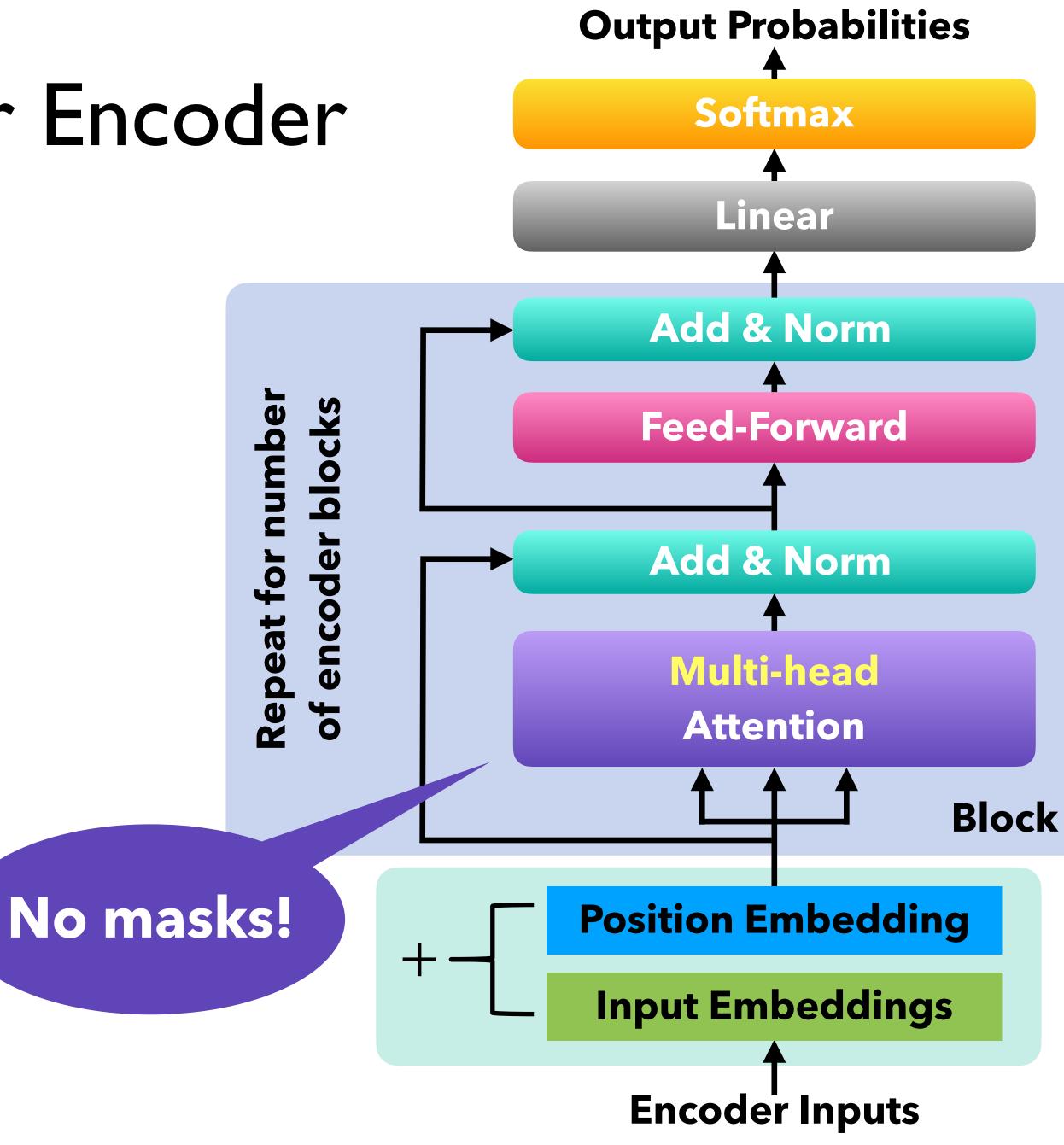
The Transformer Decoder

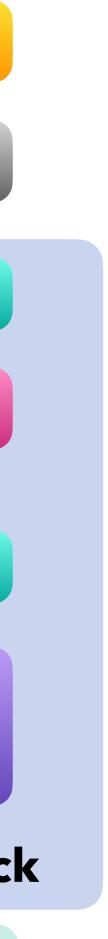
- The Transformer Decoder is a stack of Transformer Decoder **Blocks**.
- Each Block consists of:
 - Masked Multi-head Self-attention
 - Add & Norm
 - Feed-Forward
 - Add & Norm



The Transformer Encoder

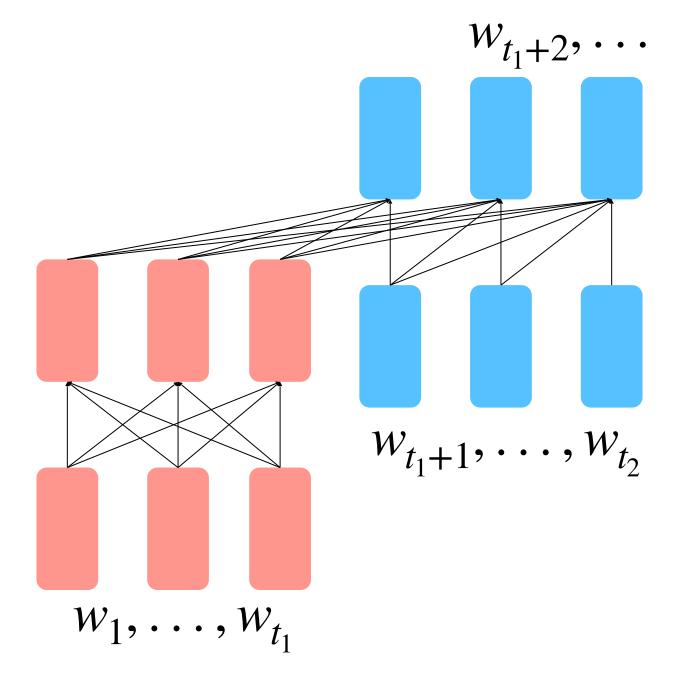
- The Transformer Decoder
 constrains to unidirectional
 context, as for language
 models.
- What if we want bidirectional context, like in a bidirectional RNN?
- We use Transformer Encoder the ONLY difference is that we remove the masking in selfattention.

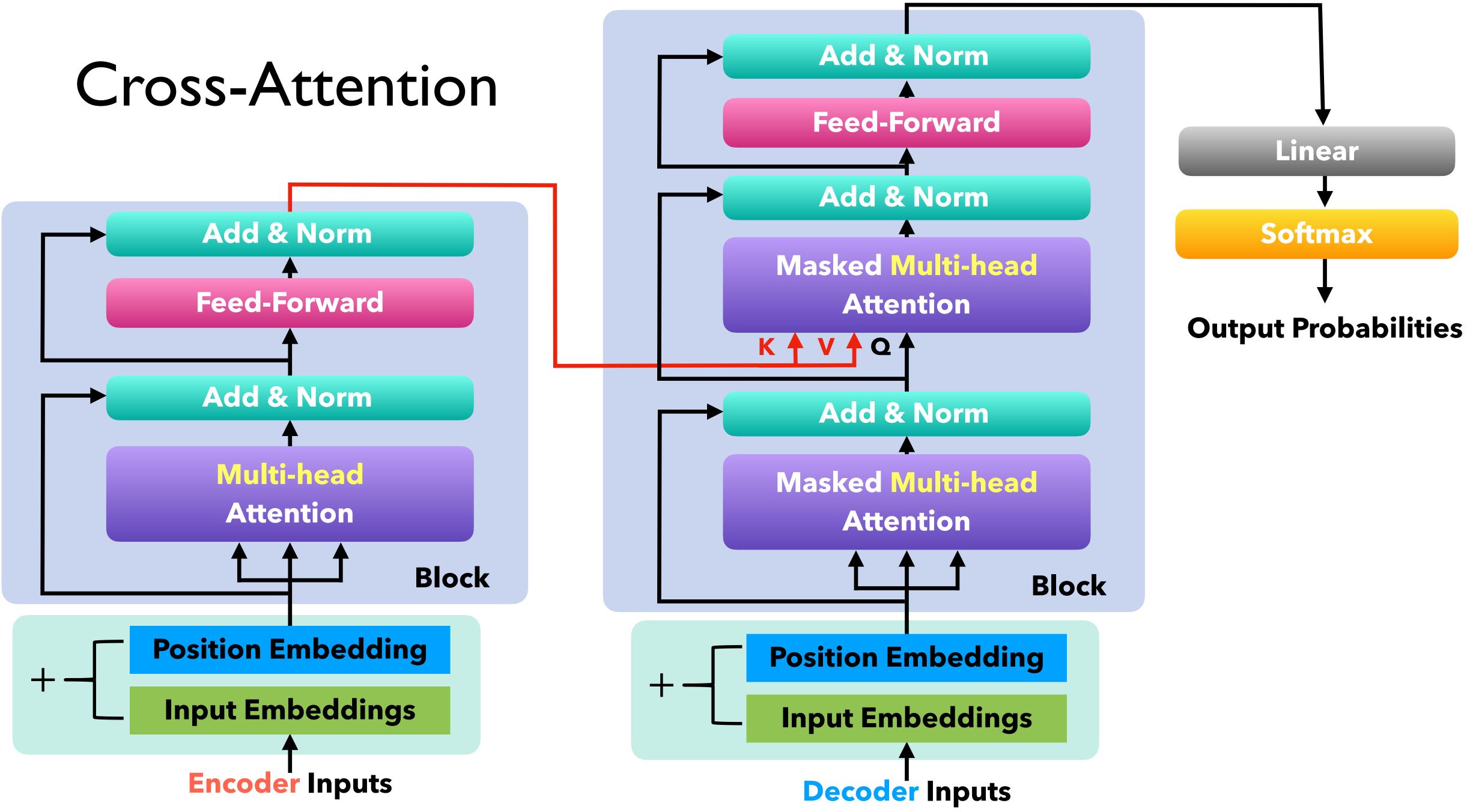




The Transformer Encoder-Decoder

- More on Encoder-Decoder models will be introduced in the next lecture!
- Right now we only need to know that it processes the source sentence with a **bidirectional** model (Encoder) and generates the target with a unidirectional model (Decoder).
- The Transformer Decoder is modified to perform **cross-attention** to the output of the Encoder.







Cross-Attention Details

- Self-attention: queries, keys, and values come from the same source.
- Cross-Attention: keys and values are from Encoder (like a memory); queries are from **Decoder**.
- Let h_1, \ldots, h_n be output vectors from the Transformer **encoder**, $h_i \in \mathbb{R}^d$.
- Let z_1, \ldots, z_n be input vectors from the Transformer decoder, $z_i \in \mathbb{R}^d$.
- Keys and values from the encoder:
 - $k_i = W_K h_i$
 - $v_i = W_V h_i$
- **Queries** are drawn from the **decoder**:

• $q_i = W_Q z_i$



Transformers: pros and cons

- \bullet
- **Easier to parallelize:**

$$\begin{aligned} Q &= XW^Q & K = XW^K & V = XW^V \\ & \text{Attention}(Q, K, V) = \text{softmax}(\frac{QK^T}{\sqrt{d_k}})V \end{aligned}$$

- Are positional encodings enough to capture positional information? Otherwise self-attention is an unordered function of its input
- **Quadratic computation in self-attention**

Can become very slow when the sequence length is large

Easier to capture long-range dependencies: we draw attention between every pair of words!

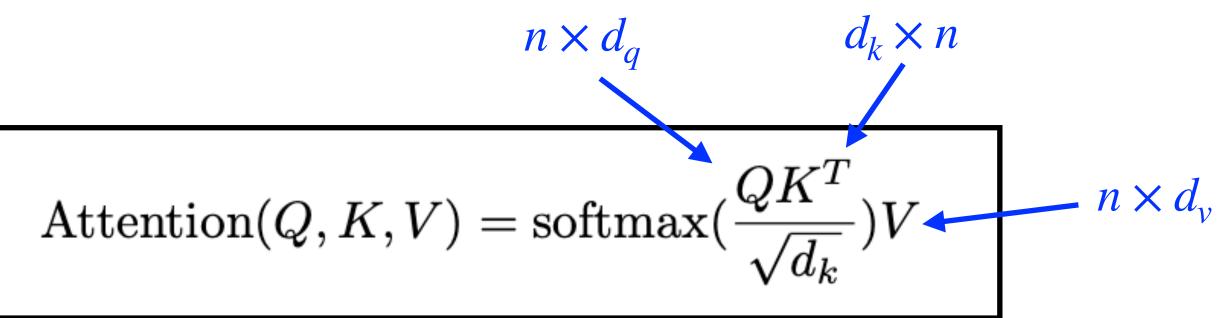
Quadratic computation as a function of sequence length

 $Q = XW^Q \qquad K = XW^K \qquad V = XW^V$

Need to compute n^2 pairs of scores (= dot product) $O(n^2 d)$ RNNs only require $O(nd^2)$ running time: $\mathbf{h}_t = g(\mathbf{W}\mathbf{h}_{t-1} + \mathbf{U}\mathbf{x}_t + \mathbf{b})$

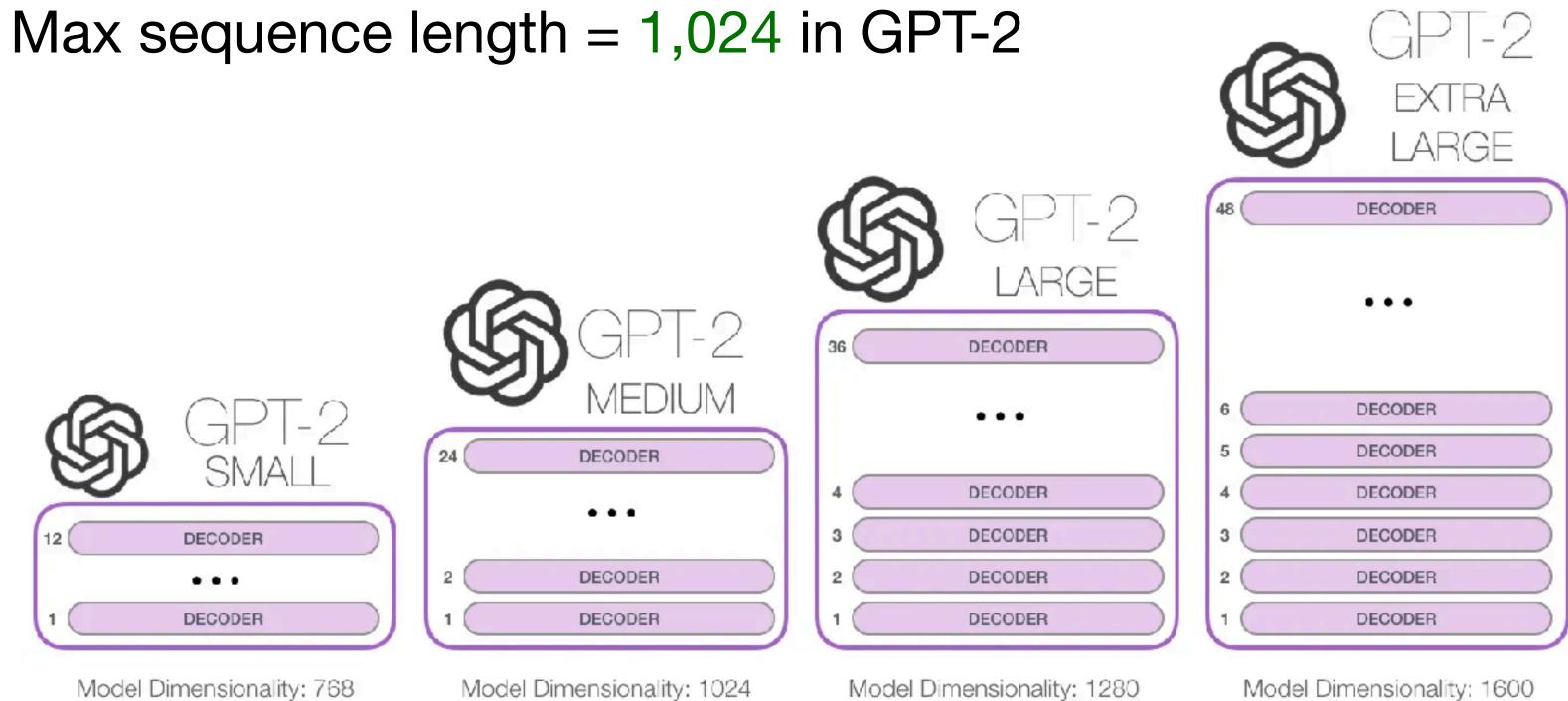
(assuming input dimension = hidden dimension = d)





Quadratic computation as a function of sequence length

Need to compute n^2 pairs of scores (= dot product) $O(n^2 d)$



What if we want to scale $n \ge 50,000$? For example, to work on long documents?

The Revolutionary Impact of Transformers

- E.g., GPT1/2/3/4, T5, Llama 1/2, BERT, ... almost anything we can name
- Transformer-based models dominate nearly all NLP leaderboards.
- Since Transformer has been popularized in language applications, computer vision also adapted Transformers, e.g., Vision Transformers.

What's next after **Transformers?**

• Almost all current-day leading language models use Transformer building blocks.

